

Computational Study of Field-Dependent Pinning Force for Pure MgB₂ Superconductor

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Abstract

The logarithmic exponential regression model is proposed for the pure MgB₂ superconductor in this study. The predicted values of normalized pinning force as a function of the reduced field are evaluated and compared to the measured values at a wide range of temperatures. The proposed model well described the best-suited model for the estimated predicted values. The predicted values of the reduced field dependence of normalized pinning force are in good agreement with the measured values for the pure MgB₂ superconductor. Some statistical error estimations namely mean bias error, mean absolute bias error and root mean square error is calculated at different temperatures from 10–30K. The large value of the coefficient of regression, R² confirms the effectiveness and significance of the proposed model. The proposed model is applicable in this study to find the accurate and precise values of the computational data.

Keywords: MgB₂; empirical model; pinning force; statistical errors

1. Introduction

In recent twenty years, several contributions are made to enhance the performance of the MgB₂ superconductor. The transition temperature, T_c of the MgB₂ is higher than the traditional Nb-based superconductors [1]. The MgB₂ continues to attract the interest of scientists due to its

remarkable properties. [2]. The high trapped field applications such as high energy particle accelerators, high field magnets, and magnetic resonance imaging require the bulk material of high critical current density, J_c at high magnetic fields [3–5]. The main disadvantage of the MgB₂ material is its low irreversibility, H_{irr} . Therefore, special efforts are needed to enhance its

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superconducting properties, especially the critical current density, J_c in high magnetic fields.

The flux pinning force is an important physical factor that can be measured by the product of critical current density, J_c , and applied field, H . Therefore, the study of the pinning force in bulk MgB₂ is significant to drive the information on the pinning mechanism [6].

Dew-Hughes study the normalized volume pinning force as a function of the applied field to analyze the behavior of the pinning mechanism in the bulk material [7]. Furthermore, Fietz and Webb proposed the one power scaling law for normalized pinning force density to recognize the nature of pinning boundaries [8]. Pinning force scaling force analysis was performed in pure MgB₂ samples with a wide range of temperatures including a spark plasma sintered one [9]. The elementary pinning force of grain boundaries is calculated using the electron scattering process and the residual resistivity [10]. In superconducting polycrystalline MgB₂, electrical connectivity and flux pinning strength of grain boundaries are significant elements to find the critical current density by the Matsushita and co-workers [10]. Extrinsic two-dimensional flux pinning centers have been created into MgB₂ superconductors using in-situ and diffusion sintering processes, with graphene-encapsulated boron powder [11]. In our previous work, I have proposed three empirical regression models including linear, exponential, and quadratic regression models that predict the transition temperature for numerous elements doped MgB₂ superconductors based on the two descriptors namely electrical connectivity index and valence energy level connectivity index [12].

Various thin films and single crystals of MgB₂ samples were prepared by spark-plasma sintering to evaluate the sharp narrow peak in volume pinning force vs. reduced field [13]. The pinning force scaling changes from grain boundary pinning to point pinning on increasing preparation temperature and finally the sharp, low-field peak in volume pinning force (F_p) disappears. The pinning behaviors of four different thicknesses for MgB₂ thin films have been discussed by Yang and co-workers [14]. These films were prepared by a hybrid physical-chemical vapor deposition technique. It was found that MgB₂ films had different growth modes in different growth stages. The effect of hydrostatic pressure up to 1.2 GPa on the critical current density and the nature of the pinning mechanism in MgB₂ has been investigated by collective theory [15]. It was found that the pressure increases the anisotropy and reduces the coherence length that resulting in weak interaction of the vortex cores with the pinning centers. The critical current density, J_c was improved in bulk pure MgB₂ samples by optimizing the sintering conditions [16]. These results reveal a strong relationship between the microstructure and the pinning performance. Thermally activated relaxation and pinning in spark-plasma sintered MgB₂ superconductor were discussed [17]. Vortex pinning was investigated with the help of field dependence of the pinning force density that indicates collective pinning by normal point-like defects. The field dependence of the pinning force in different, high-density sintered samples was analyzed for MgB₂ samples [18]. Within the grain boundary pinning mechanism, these samples show the most aspects of the field dependency of the critical force that fit the Dew-Hughes scaling law predictions.

In the present study, the normalized pinning force density, $F_p/F_{p,max}$ as a function of reduced field,

H/H_{irr} have been discussed with a wide range of temperatures using empirical models for pristine MgB_2 superconductors. The measured $F_p/F_{p,max}$ are compared with predicted ones using some statistical tools to calculate the error estimations Mean Bias Error (MBE), Mean Absolute Bias Error (MABE), and Root Mean Square Error (RMSE) included with Coefficient

of Regression (R^2). These error estimations were used to determine the least error at different temperatures 10, 15, 20, 25, and 30 K.

2. Proposed modeling

The ground boundary pinning prevails in the pure MgB_2 material, as discussed in the previous report [19]. In the present study, the normalized pinning force density, $f(x)$ can be described by the following equation:

$$f(x) = A \ln x + B e^{Cx^2} \quad (1)$$

where $f(x) = F_p/F_{p,max}$ and $x = H/H_{irr}$ is the reduced field. $F_{p,max}$ is the maximum pinning force density and H_{irr} is the irreversibility field line. In addition, A, B, and C are the empirical constants.

2.1 Proposed empirical model

The logarithmic exponential regression model (LERM) attempts to model the correlation between two variables by fitting the linear combination of logarithmic function and exponential function to observe the data. One variable is considered to be an explanatory variable, and the other is to be a dependent variable. LERM supports and predicts the behavior of the given data. This is the dominant type of regression analysis to study rigorously and used extensively in practical applications.

2.2 Statistical error estimation

In the present study, the empirical model LERM is proposed. This model estimates the statistical error to

the wide range of temperatures. Furthermore, the high R^2 value demonstrates the efficacy of the proposed model. The following are descriptions of the total sum of squares (SST) and the sum of squares of regression (SSR):

Total sum of squares

$$(SST) = \sum_{i=1}^n (\overline{Y_{cm}} - Y_{cm}^i)^2 \quad (2)$$

Sum of squares of regression

$$(SSR) = \sum_{i=1}^n (Y_{cm}^i - Y_{cp}^i)^2 \quad (3)$$

here, Y_{cm} denotes the measured value and Y_{cp} indicates the predicted one, while $\overline{Y_{cm}}$ is the mean of measured value as described underneath:

$$\overline{Y_{cm}} = \frac{1}{n} \sum_{i=1}^n Y_{cm}^i \quad (4)$$

$$\text{Coefficient of Regression } (R^2) = \frac{SSR}{SST} \quad (5)$$

The coefficient of regression (R^2) illustrates how effective the best-fit is all over the data, as described by Eq (5). The empirical model with the best fit is one with a large R^2 value near 1.

The residuals are the variations between the observed and predicted responses. It might be considered as the fundamentals of variation that the fitted model fails to explain. Residues, on the other hand, evaluate the

random errors. As a result, the random residuals support the fit of the proposed model, whereas the regular pattern displays the least-fit model. The RMSE, MBE, and MABE are some of the statistical error estimation methods used in this study as mentioned below:

$$\text{MBE} = \frac{1}{n} \sum_{i=1}^n (Y_m^i - Y_p^i) \quad (6)$$

$$\text{MABE} = \frac{1}{n} \sum_{i=1}^n |Y_m^i - Y_p^i| \quad (7)$$

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^n (Y_m^i - Y_p^i)^2} \quad (8)$$

3. Results and discussion

In the proposed model, I have fitted the measured values at different temperatures ranging from 10–30K for a pure MgB₂ superconductor. In this work, I have estimated the statistical errors by the SPSS program. The normalized pinning force density, $F_p/F_{p,max}$ vs. the reduced field, H/H_{irr} measured values for pure MgB₂ superconductor are taken out from my previous report [19]. The detailed procedure of the synthesis method is reported there. The critical current density, J_c was calculated from the width of the hysteresis loop by using Bean's model as mentioned in Ref [19]. In the present work, Fig. 1(a)–(e) elucidates the plot of normalized pinning force density, $F_p/F_{p,max}$ vs. the reduced field, H/H_{irr} at different temperatures 10, 15, 20, 25, and 30K. The measured values of $F_p/F_{p,max}$ vs. H/H_{irr} plots are fitted well with predicted values with a wide range of temperatures as shown in figures 1(a)–(e). It is remarkable from all the plots that the $F_p/F_{p,max}$ values are enhancing sharply with the increase of H/H_{irr} values thereafter, these are decreasing exponentially.

The RMSE statistic is for short-term performance, whereas the MBE figure out for the long-term. In case MBE is positive, it suggests that the predicted data is over-approximated, whereas if MBE is negative, it shows that the predicted data is under-approximated. MABE denotes the degree to which the fitting model is effective. The proposed empirical model with the least MBE, MABE, and RMSE values demonstrates the best-fitted model to the given set of data with a wide range of temperatures.

Therefore, I can accomplish that the measured values are in agreement with the predicted one for the proposed model.

The proposed model estimates some statistical errors MBE, MABE, and RMSE, among other statistical coefficients of regression, as indicated in Table 1. The evaluated coefficients of regression (R^2), are almost similar values 0.989 for all temperatures except 10K as shown in Table 1. These values of R^2 are very close to 1 that demonstrating the best-fitted values from 15–30K. The least values of statistical errors MBE, MABE, and RMSE are evaluated -0.0067, 0.02656, and 0.0346, respectively at 30K. While the highest values of MBE, MABE, and RMSE are obtained -0.0153, 0.04770, and 0.0655, respectively at a single temperature 10K as shown in Table 1. The empirical constants A, B, and C are the lowest which consists of the values 0.172, 1.422, and -2.799, respectively at 10K. I have estimated the highest peak values of H/H_{irr} 0.11765, 0.13046, 0.13447, 0.14567 and 0.16003 at 10K, 15K, 20K, 25K

and 30K, respectively for predicted fitted data. It is evident that these peaks are shifting towards the high reduced field values with the increase of temperature which is in good agreement with the previous report [19]. Table 1 shows that the proposed model best described the fitted values at 30K temperature as

4. Conclusions

I have estimated the predicted values for normalized pinning force density, $F_p/F_{p,max}$ vs. the reduced field, H/H_{irr} plot for pure MgB₂ superconductor between temperatures range 10–30K. These predicted values are in agreement with the measured values that best described the proposed model. Furthermore, the evaluated statistical errors MBE, MABE, and RMSE illustrate the least values which clearly shows the best-

compared to other temperatures. In addition, the measured values of $F_p/F_{p,max}$ vs. H/H_{irr} plots are in agreement with the predicted values for the proposed model at almost all the temperatures for the pure MgB₂ superconductor.

fitted values at 30K temperature rather than the other temperatures. The coefficient of regression, R^2 is very close to 1 at almost all temperature ranges reveals the relevant proposed model. It is obvious from all of the plots that the $F_p/F_{p,max}$ values improve abruptly with the increase of H/H_{irr} values, and then decreases exponentially afterward.

Table 1: Some empirical constants with their statistical values from temperature range 10–30K

T (K)	Empirical constants			Statistical values			
	A	B	C	MBE	MABE	RMSE	R^2
10	0.172	1.422	- 2.779	-0.0153	0.04770	0.0655	0.971
15	0.295	1.688	-2.978	-0.0081	0.03185	0.0361	0.989
20	0.312	1.732	-3.052	-0.0069	0.03187	0.0365	0.989
25	0.325	1.745	-3.055	-0.0076	0.03140	0.0370	0.989
30	0.327	1.719	-2.799	-0.0067	0.02656	0.0346	0.989

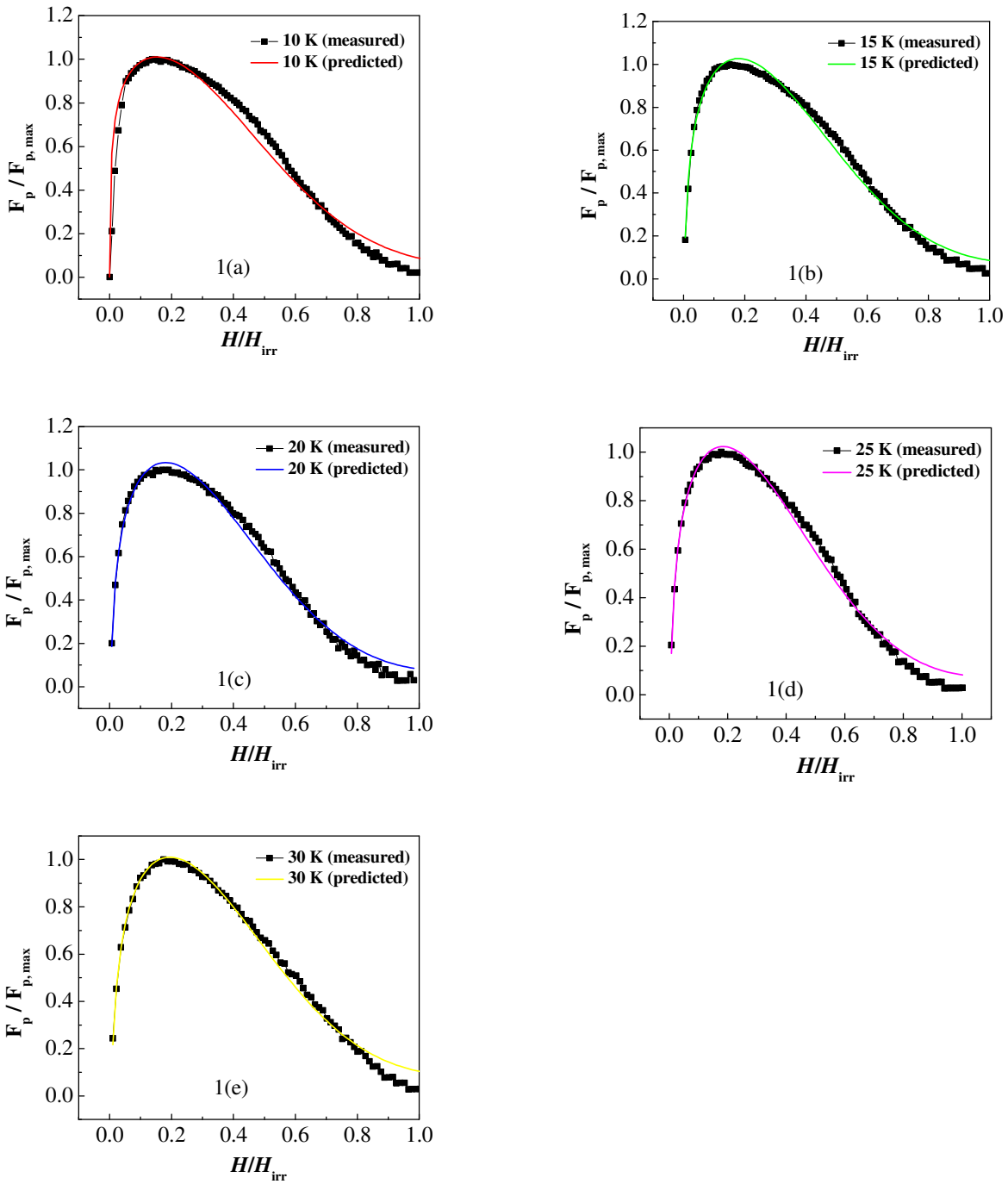


Figure 1(a)-(e): The normalized pinning force density, $F_p/F_{p,max}$ vs. the reduced field, H/H_{irr} plot for pure MgB₂ superconductor at temperature 10-30K.

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Conflicts of Interest

The author declares no conflict of interest.

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Data availability

The raw/processed data required to reproduce these findings cannot be shared at this time due to legal or ethical reasons.