Comprehensibility and Representation of the Maxima and Minima for the functions of Multi-Variables

V Naganjaneyulu[1], Shaik Mohammed Ali[2]

1,2 : Associate Professor, Lords Institute of Engineering and Technology, Hyderabad.

Abstract. The formal definition of the concept of maxima and minima, due to its dynamic essence, is perfectly understandable by visual representation through software tools. This paper presents the classical teaching approach supported by Geogebra, for teaching and learning very specific and subtle criteria which distinguish concept maxima and minima of functions of multi variables compared to the concept of maxima and minima of function of single variable. Without this software tool, in the graphic terms, it would not be possible that students make the distinction between sophisticated class maxima and minima of multivariable and the maxima and minima of single variable. Also, the contribution of this paper is presentation some specifics examples to better understanding the concept of maxima and minima of the function pertaining to single and multi variables at the college levels on the interactive and visual way.

Keywords: Geogebra, computer-assisted instruction, maxima and minima.

Introduction

The representation process includes the use of different models for organizing, memorizing and exchanging of mathematics ideas with the aim of solving its problems and for a better interpretation of mathematics.

Representation, graphic and otherwise, in the function of reasoning has been explored by many researchers. Research on learning with representations has shown that when students interact with an appropriate representation their performance is enhanced. Computer algebra systems can be used to change the emphasis on learning and teaching of calculus away from symbolic techniques and methods towards higher-level cognitive skills that focus on concepts and problem-solving. ([1, 2, 3, 4, 6, 7,11]).

Mathematical concepts, ideas, methods, have significant support in technology that is intuitively representative in different ways. The use of them is very beneficial in the process of learning and teaching when solving problems and doing research. The research literature confirms that technology can enhance students' understanding of mathematics concepts ([1, 3, 4, 8, 10,11]) and improve their achievement. Learning calculus has been subject of extensive research for a long time. The research literature indicates that students have experienced cognitive difficulties in understanding the concept of differentiation , maxima and minima which is the key concept of mathematical analysis ([1, 2, 3]).

Visualization of Maxima and Minima of Function

In the calculus, where maxima and minima of functions is one of the core concepts, definition of maxima and minima should cover two ideas: Graphically, the graph of *the function* which is required to find extrema subject to some condition or without constraints and the second, the stationary of the critical points of the functions values of a function.

Formally a function is said to have a relative maximum at some point if it possess the greatest value in the neighborhood of that point, similarly it attains relative minimum if it has lowest value in the neighborhood of that point. Generally the point of maxima or minima is called extremum. Algebraically for the function of single variable $y=f(x)$ differentiating the function and equating to zero yields set of critical or the stationary values that is considered as necessary condition for maxima and minima. Differentiating the same function second time and checking the positivity or the negativity of the resultant value decides minimum or maximum point.

That is $f^{11}(x) > 0$ gives point of minimum. Where as $f^{11}(x) < 0$ gives point of maximum.

For example:

$f(x) = 0.93x^3-2x^2-3x+3$

f has maximum at x=-0.54 and maximum value is 3.84 and

f has minimum at x=1.98 and minimum value is -3.56

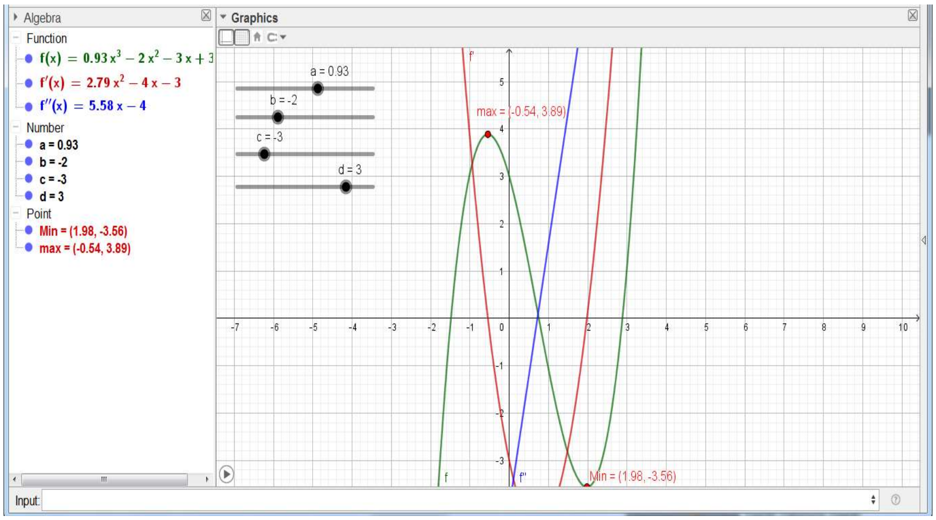

Fig 1: Finding Max/Min of $f(x) = 0.93x^3-2x^2-3x+3$

The same problem can be solved by using geogebra tool in an effective manner and graphically representation for maximum and minimum can be visualized from different views. Instead of defining the specific function

$f(x) = 0.93x^3-2x^2-3x+3$

The family of curves can be defined as

$f(x) = ax^3+bx^2+ cx+d$

By assigning different values to the parameters a,b,c, and d the maxima and minima of family of curves can be illustrated

For the above defined function in the example

a = 0.9 ,b = -2 , c = -3 and d = 3

The above figure depicts maxima and minima of the function $f(x)=0.93x^3-2x^2-3x+3$ graphically.

The following is the figure representing maxima and minima of the function $f(x)=ax^3+bx^2+ cx+d$ where a = 1.3, b = -1.4, c = -2.2, d = 3

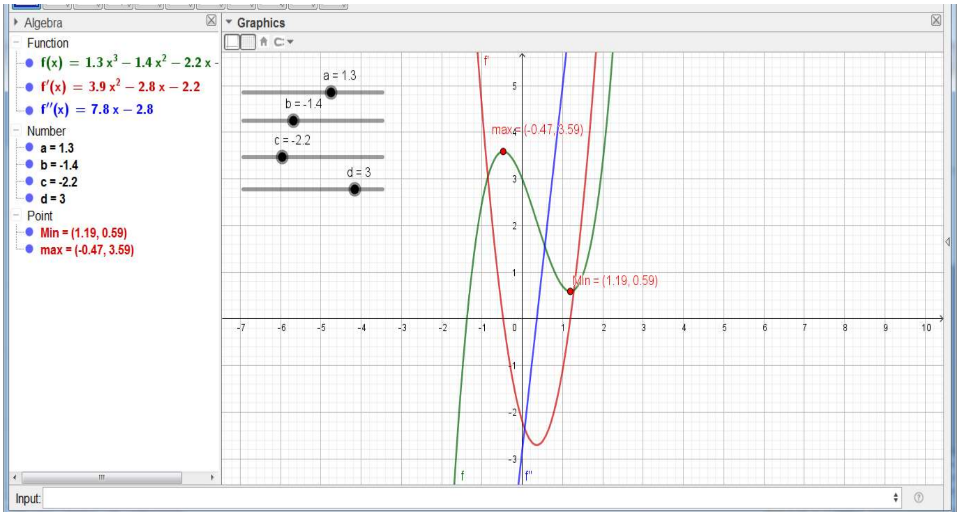

Fig 1: Finding Max/Min of $f(x) = 1.3x^3-1.4x^2-2.2x+3$

In case of function of two variables, geometrically $z=f(x,y)$ represents a surface in three dimensional space. The maximum is a point on the surface (hilltop) from which the surface descends (comedown) in every direction. The minimum is the bottom of depression from which the surface ascends in every direction. Saddle point or minimax is a point where the function is neither maximum nor minimum at such a point f is maximum in one direction while minimum in another direction.

Analytically procedure for finding maxima and minima of the function of two variables is as follows :

Solving the equations $f_x=0$ and $f_y=0$ gives stationary or the critical points P of the function.

then calculating $r = f_{xx}$, $s = f_{xy}$ $t = f_{yy}$ at point P.

The P is maximum if $rt-s^2 > 0$ and $r < 0$ then f has maximum at P

The P is minimum if $rt-s^2 > 0$ and $r > 0$ then f has minimum at P

The P is a saddle point if $rt-s^2<0$ then f has neither maximum nor minimum

The failure case if $rt-s^2=0$ then further investigation is needed.

The concept can be extended to multivariable calculus and visualized from different views. The behavior of simple to complex functions can be visualized graphically and it will pave the way for the learner to grasp complete comprehensibility of the concepts through software tools. Generally as far as graphs of functions of two and three variable are concerned it is very cumbersome task to get complete understanding analytically but it can be viewed clearly with the help of software and computer assisted learning tools.

CONCLUSION:

The computer algebra system have the quality to facilitate a dynamic approach teaching and learning process and allow the students to participate in innovating and accumulate their knowledge,therefore building up geometrical and conceptual comprehensibility by providing deeper insight into the learning process with the advent of mathematical tools, abstract and complicated concepts mathematical equations can be explained with visualization in an effect manner.

The main objective of the paper was to focused on significance and importance of mathematical software in teaching and learning process. Geogebra software was used to solve some problems pertaining to maxima and minima of functions of single and it extension to two variables analytically as well as graphically. Use of mathematical tools in learning mathematical concepts not only boost the understanding of learners but also save time, avoid tedious and monotonous calculations procedures and process. In case of maxima and minima of functions of multi variables problems concept of one has to equipped with differentiation, solving equations, curve tracing, properties of curves etc local maxima and minima of functions, global or absolute maxima or minima different functions like polynomial, exponential, logarithmic, geometric or composite functions hyperbolic and special function is cumbersome task and time consuming with limited understanding manually or analytically .The problems which may be complex or simple in nature can be dealt easily with the help of mathematical tools and much more emphasize can be focused on graphical representation, nature of curves and surfaces. Local maxima and Minima of the functions can easily be visualized by simple assisting difference interval, or regions.

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