k-OUT-OF-n SYSTEM WITH REPAIR UNDER SINGLE/BULK SERVICE

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ABSTRACT

In this study, we investigate a k-out-of-n repairable system under a server that provides single/bulk service to the failed components of the system. The components are identical in nature. The server initiates the repair of failed components upon the accumulation of N ($1 \leq N \leq$ n-k) units. These N units are taken as a group for service, and once the group service is completed, the server transitions to single service. The server is switched off only when all the failed components of the system become functional. Once turned off, the server is reactivated only upon the accumulation of N units. The time between consecutive failures and the repair times are assumed to follow independent exponential random variables. We perform steady-state analysis by examining a continuous-time Markov chain and obtain several system characteristics, including the joint probability distribution of the number of failed components and server state, system reliability, average number of failed components, distribution of the busy period of the server, expected busy period, etc. We also analyze a cost function associated with the system, which is shown to be convex in N. A comparison is made with the results obtained under a single unit of service under N-policy numerically, and it is found that the single/bulk service model yields better results. Numerical illustrations are also provided.

Key Words: Reliability, k-out-of-n: G system, Markov chain, N-policy

1. INTRODUCTION

In this study, we consider a k-out-of-n: G repairable system with identical components under a single/bulk service rule. A single server repairs the failed units of the system once activated. The server is activated when the number of failed components of the system reaches a level N $(2 \leq N \leq n-k)$. Once activated, the server takes all N waiting units as a group for service, and the remaining failures during the bulk service period are addressed with single service on a first-come, first-served (FCFS) basis.

We assume varying service rates for both single service and group services. The server remains active until all failed units are repaired, then it switches off and activates again as soon as the number of failed components accumulates to N. This process continues.

Several researchers have investigated Npolicy for queues. In a queueing system under N-policy, the server is on vacation until N units accumulate for service for the first time after becoming idle. As soon as N units accumulate in the system, the server starts service, one at a time, until the system becomes empty. The server is turned on again when the queue size reaches the number N. The process continues in this fashion.

For a repairable reliability system, it is natural to investigate when a repairman should be called. Hiring a repairman on the failure of a single unit and having them leave as soon as all subsequent repairs are completed results in frequent activations. This is compared to the case where the repairman is activated on the failure of N units. Similarly, calling the server on the accumulation of a large number of failed units may lead to heavy losses, as the system reliability is significantly reduced.

In the context of the reliability of k-out-of-n: G systems with repair, Krishnamoorthy et al. (2002) addressed the problem of determining the accumulated number of failed components to call the repairman, aiming to minimize running costs and maximize system reliability. In our paper, we extend this work by introducing a bulk service rule to the accumulated N failed components and providing single service to the remaining failed components during the bulk service time. This is done with varying service rates on an FCFS basis, thereby reducing waiting time costs.

These systems find direct applications in reliability engineering, production systems, satellite communications, etc. A probabilistic study of these systems aids in developing optimal strategies for maintaining high system reliability. The literature on such studies is extensive, with notable contributions from Angus (1988), Godbole et al. (1988), Pharm and Upadhyaya (1988), Chakravarthy et al. (2001), Krishnamoorthy and Ushakumari (2001), Ushakumari and Krishnamoorthy (2004), as well as Ushakumari (2018, 2021)

This paper is organized as follows: Section 2 describes the mathematical model and analysis, and in Section 3, we compute the performance measures of the system. Section 4 discusses an optimization problem and provides numerical illustrations.

2. MATHEMATICAL MODEL AND ANALYSIS

2.1 Preliminaries and notations

In the system under study, we assume that the time between consecutive failures is a random variable following an exponential distribution with parameter λ. The server initiates service when the number of failed components accumulates to N, and these N units are taken as a group for service. During the bulk service time and subsequently, if any additional failures occur, the server provides single service on a first-come, first-served (FCFS) basis. The server switches off only when there are no failed components in the system and reactivates as soon as the number of failed components accumulates to N. This process continues.

We also assume that the service times are independent exponentially distributed random variables with a service rate $\mu_1$for single service and a service rate μ_2 for group service, where $\mu_2 < \mu_1$. In addition, we assume that the time between successive failures and maintenances are independent of each other, and the repaired units are as good as new.

Let $N(t)$ and $S(t)$ denote respectively the number of failed components of the system and state of the server at time t and $S(t)$ is defined by

$$S(t) = \begin{cases} 0, \text{ if the server is idle at time t} \\ 1, \text{ if the server is under single repair at time t} \\ 2, \text{ if the server is under bulk repair at time t} \end{cases}$$

Let $Y(t) = (N(t), S(t))$. Then the process $\{Y(t), t \geq 0\}$ forms a Markov process on the state space $E = \{(i,0)/1 \leq i \leq N-1\} \cup \{(i,1)/1 \leq i \leq n-k+1\} \cup \{(i,2)/N \leq i \leq n-k+1\}$

Let $P_{ij}(t) = P\{\big(N(t), S(t)\big) = (i,j)/(N(0), S(0)) = (0,0)\}, (i,j) \in E, t \geq 0.$

2.2 STEADY STATE DISTRIBUTION OF THE SYSTEM

In this section, we derive the joint limit distribution of the number of failed components and the server state. As the Markov chain is finite and irreducible, the limit distribution exists. Let $q_{ij=} \lim_{t\to\infty} P_{ij}(t)\,, (i,j) \in E$ denote the limit distribution probability distribution.

In the steady state, the flow balance equations of the process $\{Y(t), t \geq 0\}$ are:

$$-\lambda q_{00} + \mu_1 q_{11} + \mu_2 q_{N2} = 0 \quad (1)$$

$$-\lambda q_{i0} + \lambda q_{i-10} = 0,\ 1 \leq i \leq N-1 \quad (2)$$

$$-(\lambda+\mu_1) q_{11} + \mu_1 q_{21} + \mu_2 q_{N+12} = 0 \quad (3)$$

$$-(\lambda+\mu_1) q_{i1} + \lambda q_{i-11} + \mu_1 q_{i+11} + \mu_2 q_{i+N2} = 0,\ 2 \leq i \leq n-k+1-N, i \neq N-1 \quad (4)$$

$$-(\lambda+\mu_1) q_{i1} + \lambda q_{i-11} + \mu_1 q_{i+11} = 0,\ n-k+1 < i \leq n-k \quad (5)$$

$$-(\lambda+\mu_1) q_{N-11} + \lambda q_{N-21} = 0 \quad (6)$$

$$-\mu_1 q_{n-k+11} + \lambda q_{n-k1} = 0 \quad (7)$$

$$-(\lambda+\mu_2) q_{N2} + \lambda q_{N-10} = 0 \quad (8)$$

$$-(\lambda+\mu_2) q_{N+i\,2} + \lambda q_{N+i-1\,2} = 0,\ 1 \leq i \leq n-k+1-N \quad (9)$$

We can obtain the joint limit distribution of the number of failed components and the server state from equations (1) to (8) and are as follows.
From equation (2), we have $q_{i0}= q_{i-1\,0}, for\ i = 1, 2, \dots N-1$ and from equation (8),
$$q_{N2} = \frac{\lambda}{\lambda+\mu_2} q_{N-10} = \frac{\lambda}{\lambda+\mu_2} q_{00}$$
From equation (1),$q_{11} = \frac{\lambda}{\mu_1}\frac{\lambda}{\lambda+\mu_2} q_{00}$ and from equation (9) we get
$q_{N+i\,2} = \left(\frac{\lambda}{\lambda+\mu_2}\right)^{i+1} q_{00},\ 1 \le i \le n-k+1-N$. Also from equation (3), we can obtain
q_{21} =[$\frac{\lambda}{\mu_1}\left(\frac{\lambda}{\lambda+\mu_2}\right)^2 + \left(\frac{\lambda}{\mu_1}\right)^2 \frac{\lambda}{\lambda+\mu_2}$] q_{00}. Now equation (4) can be written as
$$q_{i+11} - \left(1+\frac{\lambda}{\mu_1}\right) q_{i1} + \frac{\lambda}{\mu_1} q_{i-1\,1} = -\frac{\mu_2}{\mu_1}\left(\frac{\lambda}{\lambda+\mu_2}\right)^{i+1} q_{00} \qquad (10)$$
which is a non-homogeneous difference equation. To solve equation (10), use the right shift operator E such that$E q_{i1} = q_{i+1\,1}$. Then from (10), we get
$[E^2 - \left(1+\frac{\lambda}{\mu_1}\right) E + \frac{\lambda}{\mu_1}] q_{i1} = -\frac{\mu_2}{\mu_1}\left(\frac{\lambda}{\lambda+\mu_2}\right)^{i+1} q_{00}$, is a non-homogeneous equation of order 2. Let $r(E) =$ $[E^2 - \left(1+\frac{\lambda}{\mu_1}\right) E + \frac{\lambda}{\mu_1}$. Solution of the homogeneous part of the equation is $A(\frac{\lambda}{\mu_1})^i + B$, where A and B are arbitrary constants. Hence the general solution of the equation is
$$q_{i1} = A(\frac{\lambda}{\mu_1})^i + B + \frac{-\frac{\mu_2}{\mu_1}\left(\frac{\lambda}{\lambda+\mu_2}\right)^{i+1} q_{00}}{r(\frac{\lambda}{\lambda+\mu_2})}, \qquad 2 \le i \le n-k+1-N, i \ne N-1.$$
Since$\sum_{i=1}^{n-k+1-N} q_{i1} < 1$, we get $B = 0$. Now choose A in such a way that the results hold for $i = 1$. Then we get
$q_{i1} = [\left(\frac{\lambda}{\lambda+\mu_2} + \frac{\mu_1}{\lambda+\mu_2-\mu_1}\right)\left(\frac{\lambda}{\mu_1}\right)^i - \frac{\lambda}{\lambda+\mu_2-\mu_1}\left(\frac{\lambda}{\lambda+\mu_2}\right)^{i-1}] q_{00}, 2 \le i \le n-k+1-N, i \ne N-1$. Also, we have from (6)
$$q_{N-1\,1} = \frac{\lambda}{\lambda+\mu_1} q_{N-2\,1}$$
$$= \frac{\lambda}{\lambda+\mu_1}\left[\left(\frac{\lambda}{\lambda+\mu_2} + \frac{\mu_1}{\lambda+\mu_2-\mu_1}\right)\left(\frac{\lambda}{\mu_1}\right)^{N-2} - \frac{\lambda}{\lambda+\mu_2-\mu_1}\left(\frac{\lambda}{\lambda+\mu_2}\right)^{N-3}\right] q_{00}$$
Also using equations (5) and (7), we can write
$q_{i1} = \frac{\lambda}{\mu_1} q_{i-1\,1},\quad n-k+1-N < i \le n-k$. Now, we can write q_{i1}recursively as
$q_{i1} = \left(\frac{\lambda}{\mu_1}\right)^{i-(n-k+1-N)} q_{00},\quad n-k+1-N < i \le n-k$ and
$q_{n-k+1\,1} = (\frac{\lambda}{\mu_1})^N q_{n-k+1-N\,1}.q_{00}$ can be computed using the condition
$\sum_{i=0}^{N-1} q_{i0} + \sum_{i=1}^{n-k+1} q_{i1} + \sum_{i=0}^{n-k+1-N} q_{N+i\,2} = 1$. Thus, we have the following:

<u>Theorem1</u>: The joint limit distribution of the number of failed components and the server state are given by
$q_{i0} = q_{i-1\,0}, 1 \le i \le N-1$
$$q_{N+i\,2} = \left(\frac{\lambda}{\lambda+\mu_2}\right)^{i+1} q_{00},\ 0 \le i \le n-k+1-N$$
$$q_{11} = \frac{\lambda}{\mu_1}\frac{\lambda}{\lambda+\mu_2} q_{00}$$
$q_{i1} = [\left(\frac{\lambda}{\lambda+\mu_2} + \frac{\mu_1}{\lambda+\mu_2-\mu_1}\right)\left(\frac{\lambda}{\mu_1}\right)^i - \frac{\lambda}{\lambda+\mu_2-\mu_1}\left(\frac{\lambda}{\lambda+\mu_2}\right)^{i-1}] q_{00},\ 2 \le i \le n-k+1-N, i \ne N-1$
$$q_{N-1\,1} == \frac{\lambda}{\lambda+\mu_1}\left[\left(\frac{\lambda}{\lambda+\mu_2} + \frac{\mu_1}{\lambda+\mu_2-\mu_1}\right)\left(\frac{\lambda}{\mu_1}\right)^{N-2} - \frac{\lambda}{\lambda+\mu_2-\mu_1}\left(\frac{\lambda}{\lambda+\mu_2}\right)^{N-3}\right] q_{00}$$
$$q_{i1} = \left(\frac{\lambda}{\mu_1}\right)^{i-(n-k+1-N)} q_{00},\quad n-k+1-N < i \le n-k$$

$q_{n-k+1\,1} = (\frac{\lambda}{\mu_1})^N q_{n-k+1-N\,1}$ and $q_{00} = \{N + \frac{\lambda\theta}{\mu_1-\lambda}\left(1-\left(\frac{\lambda}{\mu_1}\right)^{N-2}\right) - \delta\left(1-\left(\frac{\lambda}{\lambda+\mu_2}\right)^{N-2}\right) + \frac{\lambda}{\lambda+\mu_1}\Delta_{N-2} + \theta\left(\frac{\lambda}{\mu_1}\right)^N \frac{\mu_1}{\mu_1-\lambda}\left(1-\left(\frac{\lambda}{\mu_1}\right)^{n-k+2-2N}\right) + \delta\left(\frac{\lambda}{\lambda+\mu_2}\right)^{N-1}\left(1-\left(\frac{\lambda}{\lambda+\mu_2}\right)^{n-k-2N+2}\right) + \left(\frac{\lambda}{\mu_1-\lambda}\left(1-\left(\frac{\lambda}{\mu_1}\right)^{N-1}\right) + \left(\frac{\lambda}{\mu_1}\right)^N\right)\Delta_{n-k+1-N} + \frac{\lambda}{\mu_2}\left(1-\left(\frac{\lambda}{\lambda+\mu_2}\right)^{n-k+2-N}\right)\}^{-1}$

where $\theta = \frac{\lambda}{\lambda+\mu_2} + \frac{\mu_1}{\lambda+\mu_2-\mu_1}$, $\delta = \frac{\lambda}{\lambda+\mu_2-\mu_1}\left(1+\frac{\lambda}{\mu_2}\right)$ and $\Delta_i = \theta\left(\frac{\lambda}{\mu_1}\right)^i - \frac{\lambda}{\lambda+\mu_2-\mu_1}\left(\frac{\lambda}{\lambda+\mu_2}\right)^{i-1}$.

Remark1: When the server is only undergoing single service (i.e., when μ_2 =0), this model reduces to Krishnamoorthy et al. (2002).

3. SOME IMPORTANT SYSTEM CHARACTERISTICS

In this section, we have computed several performance measures of the system, which are useful for investigating the optimal N value.

1,System reliability in the long run = $1-(q_{n-k+1\,1} + q_{n-k+1\,2})$

$$=1-[(\frac{\lambda}{\mu_1})^N\Delta_{n-k+1-N} + -\left(\frac{\lambda}{\lambda+\mu_2}\right)^{n-k+1-N}]q_{00}$$

2. Expected number of failed components in the system when server is idle

$$=\sum_{i=1}^{N-1} iq_{i0} = \frac{N(N-1)}{2}q_{00}$$

3. Expected number of failed components when the server is under bulk service $=\sum_{i=1}^{n-k+1-N} iq_{N+i\;2}$

$$= \left(\frac{\lambda}{\mu_2}\right)^2 - [(n-k+1-N)\left(1+\frac{\lambda}{\mu_2}\right) - \left(1+\frac{\lambda}{\mu_2}\right)^2]\left(\frac{\lambda}{\lambda+\mu_2}\right)^{n-k+3-N} q_{00}$$

4. Expected number of failed components when the server is under single service

$= \sum_{i=1}^{n-k+1} iq_{i1}$

$=(T_1 + T_2 + T_3 - T_4 + T_5 * T_6 + T_7)q_{00}$, where

$T_1 = \frac{\lambda\mu_1}{(\mu_1-\lambda)^2}[1-{\rho_1}^{N-1} + (N-1){\rho_1}^N$, $T_2 = \frac{N\lambda}{\lambda+\mu_1}[\theta{\rho_1}^{N-2} - \delta{\rho_1}^{N-3}]$

$T_3 = \frac{\theta\rho_1}{(1-\rho_1)^2}[(1-\rho_1)\rho^{N+1}(N+2) + \rho^{N+2} - (n-k-N){\rho_1}^{n-k-N-1}(1-\rho_1) - {\rho_1}^{n-k-N}]$

$T_4 = \delta\left(1+\frac{\lambda}{\mu_2}\right)^2 [\frac{(N+2)\mu_2\rho^{N+1}}{\lambda+\mu_2} + \rho^{N+2} - \frac{(n-k-N)\mu_{2}\rho^{n-k-N-1}}{\lambda+\mu_2} - \rho^{n-k-N}$

$T_5 = \Delta_{n-k+1-N}\frac{1}{{\rho_1}^{n-k-N}(1-\rho_1)^2}$

$T_6 =[(1-\rho_1){\rho_1}^{n-k+1-N}(n-k+2-N) + {\rho_1}^{n-k+2-N} - (1-\rho_1){\rho_1}^{n-k}(n-k+1) - {\rho_1}^{n-k+1}$, $T_7 = {\rho_1}^N\Delta_{n-k+1-N,}$ where $\rho_1 = \frac{\lambda}{\mu_1}$ and $\rho = \frac{\lambda}{\lambda+\mu_2}$

5. Probability Distribution of Busy Period of the Repairman

The probability distribution of the busy period of the repairman represents the distribution of time during which the repairman is continuously active. To compute this, consider the Markov chain over the state space $E_1 = (N,2),(N+1,2),\ldots(n-k+1,2),(n-k+1,1),\ldots,(1,1),(0,0)\}$ with (0, 0) as an absorbing state. The distribution of busy period of the server is the distribution of the time until reaching state $(0,0)$ starting from the state $(N,2)$. The transition rate matrix of this Markov chain is $Q_1 = \begin{pmatrix} T_1 & e_\mu \\ \underline{0} & 0 \end{pmatrix}$ where T_1 is a matrix obtained by deleting the last row and last column of the rate matrix and is of order $2(n-k+1)-N+1$, e_μ is a column vector with the first and last entry as μ_2 and μ_1 respectively and all other entries zero and $\underline{0}$ is the row vector of zeros of order $1x2(n-k+1)-N+1$. The distribution of time till absorption is of phase type with distribution function given by $F(x) = 1-\underline{\alpha}e^{T_1x}\underline{e}$, for $x \geq 0$ where $\underline{\alpha}$ is a row vector with 1 at the position corresponding to state $(N,2)$ and 0 at all other positions and $\underline{e}$ is a column vector of $1'$s. (For details, see Neuts (1981)). Its probability density function is given by $f(x) = \underline{\alpha}e^{T_1x}e_\mu$, $x \geq 0$.

6. Repairman's Expected Busy Period

Here we develop a recursive approach to find the expected busy period of the repairman. Let $E(B)$ be the expected time to reach $(0,0)$ starting from state$(N,2)$. We compute this by conditioning on the number of failures during the bulk service period as follows:
(i) Expected busy period of the repairman when no failures during the bulk service period.
In this case, after service completion the server is switched off and (ii) Expected busy period of the repairman given at least one failure during the bulk service period. In this case, after completing the bulk service, the repairman turns into single service and continue until all units has been served out. Now in case (i),
E (busy period of the repairman given no failures during the bulk service) $=\frac{1}{\mu_2}$
To obtain the expected busy period in case (ii), we consider a Markov chain on the state space E_1. Define T_i as the time to reach state $(i-1,1)$ from state $(i,1)$, $2 \le i \le n-k$ with $E(T_{n-k+1\,1}) = \frac{1}{\mu_1}$. Also, define the random variable
$I_i = \begin{cases} 1, \text{ if the transition is from}(i,1)\text{to } (i+1,1) \\ 0, \text{ if the transition is from}(i,1)\text{to } (i-1,1) \end{cases}$. Then we have $P(I_i = 1) = \frac{\lambda}{\lambda+\mu_1}$ and
$P(I_i = 0) = \frac{\mu_1}{\lambda+\mu_1}$. Therefore $E(T_i/I_i = 0) = \frac{1}{\lambda+\mu_1}$ and

$$E(T_i/I_i = 1) = \frac{1}{\lambda+\mu_1} + E(T_{i+1}) + E(T_i)$$

Hence, we can write $E(T_i)$ as $E(T_i) = \frac{1}{\mu_1} + \frac{\lambda}{\mu_1} E(T_{i+1}), 1 \le i \le n-k$

Recursively, we can write it as $E(T_i) = \frac{1}{\mu_1} \frac{1-(\frac{\lambda}{\mu_1})^{i+1}}{1-\frac{\lambda}{\mu_1}}$, i ≥ 1. Therefore

E (Busy period/at least one failure during bulk service) $=\sum_{i-n-N-k+1}^{n-k} E\,(T_i)=$
$\frac{1}{\mu_1-\lambda}\left[\mathrm{N} - \left(\frac{\lambda}{\mu_1}\right)^{n-N-k+2} \frac{1-\left(\frac{\lambda}{\mu_1}\right)^{N+1}}{1-\frac{\lambda}{\mu_1}}\right]$. Now probability of no failure during a bulk service period $=\frac{\mu_2}{\lambda+\mu_2}$.
Therefore the expected busy period of the server is

$$E(B) = \frac{1}{\mu_2} + \frac{\lambda}{\lambda+\mu_2}\frac{1}{\mu_1-\lambda}\left[\mathrm{N} - \left(\frac{\lambda}{\mu_1}\right)^{n-N-k+2} \frac{1-\left(\frac{\lambda}{\mu_1}\right)^{N+1}}{1-\frac{\lambda}{\mu_1}}\right]$$

7. Expected Cycle Length
The expected length of a cycle (E(C)) is the expected time for $(0,0)$ to $(0,0)$ transition. It is the sum of the expected length of the idle period of the repairman and expected busy period duration. Therefore$E(C) = \frac{N}{\lambda} + E(B)$.
8. Expected number of times the system is down in a cycle
The system will be down in a cycle if it reaches either the state (n-k+1, 1) or the state (n-k+1, 2). Hence the long run fraction of time the system is down in a cycle is $q_{n-k+1\,1} + q_{n-k+1\,2}$. Here the state $(0,0)$ is recurrent. Therefore, the expected number of visits to $(n-k+1,1)$ or $(n-k+1,2)$ between two successive returns to (0,0) is $\frac{1}{q_{00}}(q_{n-k+1\,1} + q_{n-k+1\,2})$. Hence, the expected time the system is down in a cycle $E(D) = \frac{1}{\mu_1}\frac{q_{n-k+1\,1}}{q_{00}}\frac{\lambda}{\lambda+\mu_2} + \frac{1}{\mu_2}\frac{q_{n-k+1\,2}}{q_{00}}\frac{\mu_2}{\lambda+\mu_2}$.

4. OPTIMIZATION PROBLEM
In this section, we aim to investigate the optimal value of the control variable N to start service. For this, we consider the following costs:
Let K be the fixed cost of activating the server per unit time, K_1 be the unit time service cost associated with batch service, and C be the cost per unit time due to the system being down. Then the total expected

cost per unit time (TEC) is $F(N) = \frac{K+K_1}{E(B)} + \mathrm{CE(D)}$. Substituting the values of $E(B)$ and $E(D)$, we can write $F(N) = G(N) + H(N)$, (11)

where $G(N) = (K+K_1)[\frac{1}{\mu_2} + \frac{\lambda}{\lambda+\mu_2}\frac{1}{\mu_1-\lambda}\left(\mathrm{N} - \left(\frac{\lambda}{\mu_1}\right)^{n-N-k+2}\frac{1-\left(\frac{\lambda}{\mu_1}\right)^{N+1}}{1-\frac{\lambda}{\mu_1}}\right)]^{-1}$ and

$H(N) = \frac{C}{q_{00}}[\frac{1}{\mu_1}\frac{q_{n-k+1\,1}}{q_{00}}\frac{\lambda}{\lambda+\mu_2} + \frac{1}{\mu_2}\frac{q_{n-k+1\,2}}{q_{00}}\frac{\mu_2}{\lambda+\mu_2}]$.

Theorem 2: The TEC given in (11) is convex in N and hence the global minimum N^* of N exists.

Proof. From equation (11), we observe that as N increases, G(N) decreases in N, and H(N) increases in N. Hence, the function F(N) is convex in N, and the minimum value N* of N exists and can be attained.

4.1 Numerical Illustration

Here we obtained the minimum value N^* of N by computing the total expected cost per unit time for different parameter values. As particular case, we choose $\mu_2 = \frac{\mu_1}{\mathrm{N}}$, which means that the expected service time for a batch of N unit is less than the time needed for N single services. For the given input parameters λ=1, $\mu_1 = 2.5$,$\mu_2 = \frac{\mu_1}{N}$, n=30, k=15, K=100, K1=25, C=500, we computed the total expected cost per unit time and is given in the following table 4.1.1

Table 4.1.1

N	1	2	3	4	5	6	7	8
TEC(N)	218.22	96.08	58.43	41.08	33.10	32.01	37.61	50.25
N	9	10	11	12	13	14	15	16
TEC(N)	70.17	97.29	131.28	171.59	217.55	268.43	323.53	382.19

From the table, we observe that the total expected cost per unit time decreases as N increases, reaching a minimum value, and then increases. Here, the minimum value *N∗* of N is 6, and the minimum expected cost per unit time is 32.01..

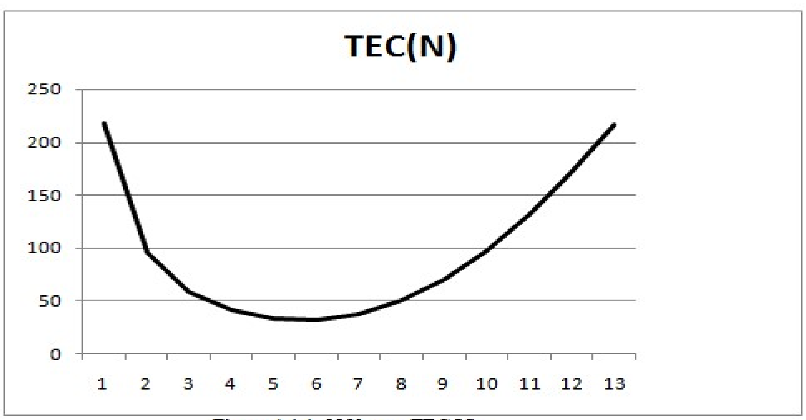

Figure 4.1.1- N Versus TEC(N)

The table 4.1.2 shows the expected number of times the system is down(E(D) in the steady state for the given parameters λ=6, $\mu_1 = 10$,$\mu_2 = \frac{\mu_1}{N}$, $n = 20$, $k = 10$. Here we can see that as N increases, $E(D)$ increases.

Table 4.1.2

N	1	2	3	4	5	6	7	8	9	10

$E(D)$	0.4	0.5	1.0	1.2	1.6	2.0	2.4	2.8	3.2	3.6

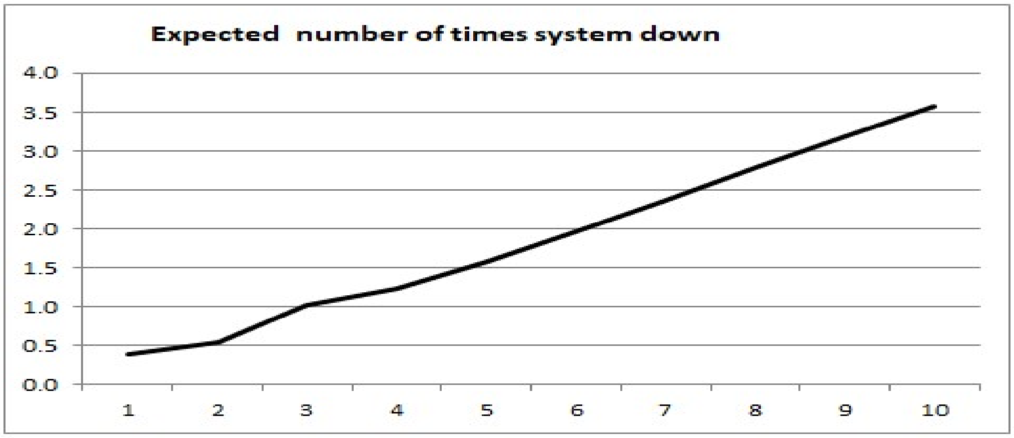

Figure 4.1.2-N versus ED

We also calculated the expected busy period of the server, expected idle period of the server, expected cycle length and expected down time in a cycle for the given parameters: λ=6, $\mu_1 = 10$, $\mu_2 = \frac{\mu_1}{N}$, n=20, k=10 and these values are presented table 4.1.3. Here we observe that as N increases, expected busy period of the server (EB), expected idle period (EI) of the server, expected cycle length (EC) and expected down time (EDT) in a cycle increases, as expected.

Table 4.1.3

N	EI	EB	EC	EDT
1	0.1667	0.1932	0.3599	0.0143
2	0.3333	0.4711	0.8044	0.0294
3	0.5	0.7786	1.2786	0.0661
4	0.6667	1.099	1.7657	0.0903
5	0.8333	1.425	2.2583	0.125
6	1	1.7517	2.7517	0.1655
7	1.1667	2.0749	3.2415	0.2097
8	1.3333	2.3888	3.7221	0.2558
9	1.5	2.6852	4.1852	0.3025
10	1.6667	2.9507	4.6174	0.3488

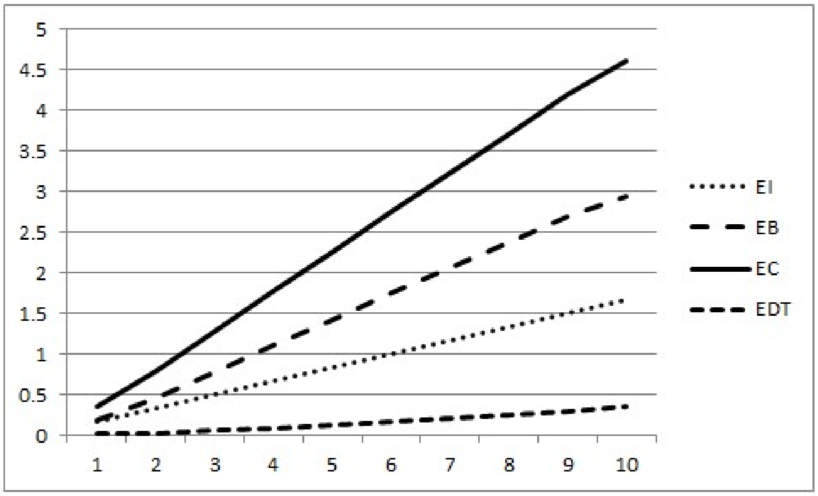

Figure 4.1.3-N-versus EB, EI, EC and EDT

5.1 Comparison of k-out-of-n system under single service and single/bulk service

We compared the system reliability of the present model with the system reliability of the single server model under the N-policy, as studied by Krishnamoorthy et al. (2002) for the given input parameters λ=2, $\mu_1 = 5, \mu_2 = \frac{\mu_1}{N}, n = 30, k = 15$ for the bulk service model and λ=2, $\mu_1 = 5, n = 30, k = 15$ for the single server model. The results are presented in table 4.1.4. Notably, the system reliability under bulk service exhibits a significant increase for each value of N.

Table 4.1.4

N	3	4	5	6	7	8	9
Reliability under single service	0.2628	0.2219	0.1827	0.1531	0.1312	0.1146	0.1017
Reliability under group service	0.9999	0.9996	0.9987	0.997	0.9945	0.9913	0.9874
N	10	11	12	13	14		
Reliability under single service	0.0914	0.083	0.076	0.0701	0.0649		
Reliability under group service	0.9828	0.9776	0.9717	0.9652	0.9582		

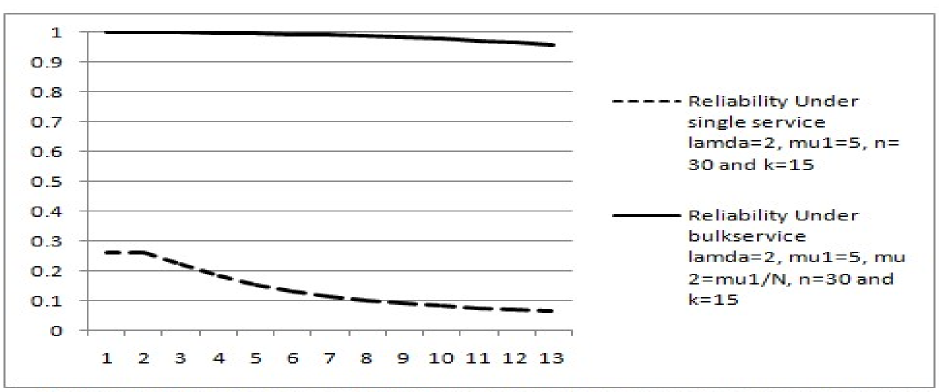

Figure4.1.4: N Versus system reliability under single service and under group service

Additionally, we have obtained the expected down time (EDT), in a cycle for the values λ=10, $\mu_1 = 8,\ \mu_2 = \frac{\mu_1}{N}, n = 30, k = 15$ for the bulk service model and λ=10, $\mu_1 = 8, n = 30, k = 15$ for the single server model. It is evident that the single/bulk service model yields better results..

Table 4.1.5

N	2	3	4	5	6	7	8	9	10
EDT under Single service	7.993	10.835	13.109	14.928	16.383	17.547	18.479	19.224	19.820
EDT under single/ group service	4.326	6.141	7.535	8.598	9.388	9.939	10.261	10.349	10.180

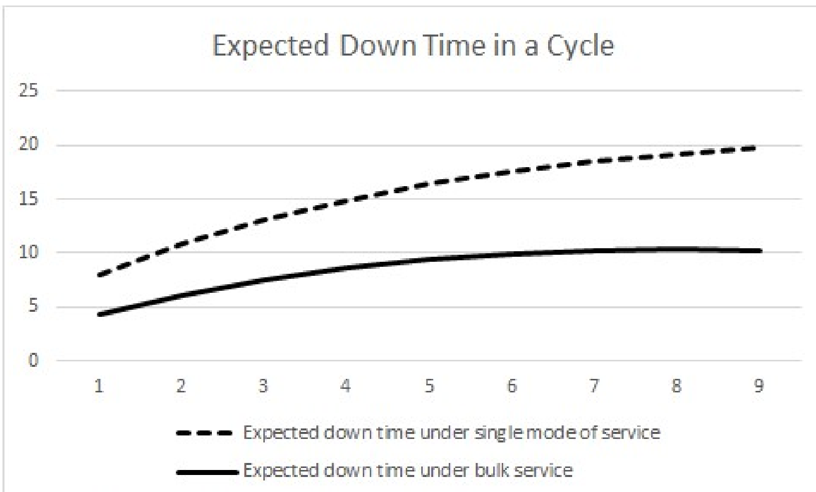

Figure 4.1.5: N versus EDT under single service and single/group service

DISCUSSIONS AND CONCLUSION

In this paper, we explore a k-out-of-n: G repairable system with identical components under single/bulk service using the N -policy. The server provides single/bulk service to the failed components of the system. The repair process initiates when the number of failed components accumulates to N ($\leq n-k$), and these N units are serviced as a group. Once the group service is completed, the server transitions to single service and is switched off only when all the failed components of the system become functional. After being switched off, the server reactivates only upon the accumulation of N units, and this process repeats.

Our primary goal was to determine the optimal value of N that minimizes the total expected cost of the system, thereby maximizing system reliability. We conducted steady-state analysis by examining a continuous-time Markov chain and obtained several system characteristics, including the joint probability distribution of the number of failed components and server state, system reliability, average number of failed components, distribution of the busy period of the server, expected busy period, etc., to achieve this goal. We also analyzed a cost function associated with the system, which was shown to be convex in N, and obtained the optimal N value to start the server. Additionally, we compared the system reliability with the results obtained under a single unit of service under N -policy, as studied by Krishnamoorthy et al. (2002), numerically and found that the single/bulk service model studied here yields better results. Furthermore, we provided a few numerical illustrations.

Future Work: The model we studied here can be extended to various other service policies such as T-policy, D-policy, etc., studied under single/bulk mode of service and under general failure time distributions and service time distributions.

REFERENCES

Angus J.E. On computing m.t.b.f. for a k-out-of-n: G repairable system, IEEE Transactions on Reliability, 37(3), 312-313, 1988.

Chakravarthy, S.R, Krishnamoorthy, A and Ushakumari, P.V. A k-out-of-n Reliability system with an unreliable server and phase type repairs and services: The (N, T) policy, Journal of Applied Mathematics and stochastic Analysis, 14(4), 361-380, 2001.

Godbole, A.P. and Potter L.K.and Sklar.J.K., Improved upper bounds for the reliability of d-dimensional consecutive k-out-of-n:f systems, Naval Research Logistics, 45(2), 219-231,1988.

A. Krishnamoorthy, A and Ushakumari, P.V. *k*-out-of-*n*: *G* system with repair: the *D*-policy, Computers and Operations Research, 28, 973–981.2001.

Krishnamoorthy A, Ushakumari, P.V and Lakshmi, B, *k*-out of *n*-system with repair: the *N*-policy Asia Pacific J. OR 19, 47–61, 2002.

Pharm. H and Upadhyaya.S, The efficiency of computing the reliability of k-out-of-n system, IEEE Transactions on Reliability, 37, 521-523, 1988.

Neuts, M.F, Matrix Geometric Solutions in Stochastic Models-An Algorithmic Approach, the John Hopkins University Press, Baltimore and London, 1981.

Ushakumari, P.V.and Krishnamoorthy, A. *k*-out-of-*n* system with repair: the max *(N, T)* policy, Performance Evaluation 57, 221–234, 2004.

Ushakumari P.V.(2018), Reliability of k out of- n system with repair and Two modes of service under N-policy, International Journal of Pure and Applied Mathematics, 119(16), 5189-5196.

Ushakumari P.V. (2021), On a Two Server Reliability System with One Server Idle Below a Threshold, International Journal of Mathematics in Operational Research, 18(4), 528-544.