DETERMINATION OF TIME EVOLUTION OF THE GRAVITATIONAL CONSTANT IN THE THEORY OF BRANS-DECKE FIELD EQUATIONS USING SIGMOID FUNCTION.

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Abstract

This study we derive sigmoid function to determine time-dependence of the dynamical gravitational constant G In the theory of Brans-Decke field equations. For this formulation, Brans-Dicke field equations for a matter-dominated, pressure-free, and spatially flat universe with homogeneous and isotropic space-time were employed. The reciprocal of the Brans-Dicke scalar field (∅) is the gravitational constant (G). The various values of a constant param can be calculated using a simple ansatz that represents the Brans-Dicke scalar field (∅) as a function of time. Using the field equations and the current values of some cosmological parameters, the potential values of a constant parameter (constituting the ansatz) have been derived. The values of that parameter (which correspond to the ansatz) lead to the conclusion that the scalar field (∅) diminishes with time, and as a result, the gravitational constant (G) rises. The value of the gravitational constant's relative time-rate of change $\left(\frac{\dot{G}}{G}\right)$ has also been calculated, and this number has been determined to be time-independent. . For all values of the parameter, belonging to the ansatz, the time-dependence of and G has been graphically illustrated. The gravitational field equations did not need to be solved, which is a novel element of this study.

Keywords: Cosmology, Brans-Dicke Theory , Sigmoid function , Scalar Field , Gravitational Constant

1. Introduction

Numerous fascinating astrophysical observations and related theoretical discoveries have spurred interest in studying alternative general relativity theories. Numerous modified gravity theories have been put forth for these kinds of investigations. [1-6]. The f(R) hypothesis and the Brans- Dicke theory stand out among them [7-9]. Brans and Dicke proposed a rema in 1961.

Roy [20] Finding the theoretical development of the time-varying gravitational constant and its relative time-rate of change with respect to the redshift parameter is the goal of the current study. Within the context of the Brans-Dicke theory of gravity, Roy [19] shows in 2021 how to calculate the time-dependence of the dynamical gravitational constant (G) mathematically in a relatively straightforward method. Based on Mach's principle, Brans and Dicke presented a very intriguing theory of gravity in 1961. Because of the way its theoretical framework was put up, the dynamics and structure of the cosmos could be used to calculate the gravitational constant.
A scalar field parameter ($\emptyset$) was introduced for this reason, causing G to behave as ($\emptyset^{-1}$).

The findings of a number of recent observations [12-16] point to the time-varying nature of G. Several cosmic events are explained by the Brans-Dicke theory in terms of a time-varying ($\emptyset$) [17, 18]. It was challenging to explain astrophysical data of accelerating cosmic expansion using the original BD hypothesis [9, 19-21].To explain the accelerated expansion of the cosmos and its transition from slowing to acceleration, a new component called dark energy was created [22]. The nature of the gravitational constant's (G) time-dependence has been determined in this study using Brans-Dicke field equations for an isotropic and homogeneous space-time in a matter-dominated world with zero pressure and spatial curvature (dust filled). The time-variation of the scalar field ($\emptyset$) has been considered to be represented by a simple ansatz in the form of a Sigmoid function of time. Using the current values of some cosmological parameters, we showed that by incorporating this sigmoid function into the field equations, $\emptyset$ is a decreasing function of time $(t = t_0)$.

The properties of time evolution of G $\left(\frac{1}{\emptyset}\right)$ have been determined without solving the differential equations involving the scale factor and its time derivatives (i.e., the field equations of the BD theory). It has been discovered that the gravitational constant is growing over time. The relative rate of change of the gravitational constant $\left(\frac{\dot{G}}{G}\right)$ has been calculated and shown to be independent of time.

2. Brans-Dicke Field Equations

For a matter-dominated, homogeneous and isotropic universe with zero spatial curvature and zero pressure (dust filled), Brans-Dicke field equations are given by

$$3\frac{\dot{a}^2}{a^2}+3\frac{\dot{a}}{a}\frac{\dot{\emptyset}}{\emptyset}-\frac{\omega\dot{\emptyset}^2}{2\emptyset^2}=\frac{\rho_m}{\emptyset} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (2.1)}$$

$$2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{\omega\dot{\emptyset}^2}{2\emptyset^2}+2\frac{\dot{a}}{a}\frac{\dot{\emptyset}}{\emptyset}+\frac{\ddot{\emptyset}}{\emptyset}=0 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (2.2)}$$

$$\frac{\ddot{\emptyset}}{\emptyset}+3\frac{\dot{a}}{a}\frac{\dot{\emptyset}}{\emptyset}=\frac{\rho_m}{(2\omega+3)\emptyset}-\frac{\dot{\omega}\dot{\emptyset}}{(2\omega+3)\emptyset} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (2.3)}$$

Where

a = Scale factor , $\dot{a}$ and $\ddot{a}$ are the 1st and 2nd derivative of a with respect to time

$\emptyset$ = Scalar field , $\dot{\emptyset}$ and $\ddot{\emptyset}$ are the 1st and 2nd derivative of $\emptyset$ with respect to time

ω = Brans-Dicke coupling parameter

ρ_m = Density of the matter

Eliminating ω term from the equations (2.1) and (2.2)

We get $4\frac{\dot{a}^2}{a^2}+2\frac{\ddot{a}}{a}=\frac{\rho_m}{\emptyset}-5\frac{\dot{a}}{a}\frac{\dot{\emptyset}}{\emptyset}-\frac{\ddot{\emptyset}}{\emptyset}$ ____________ (2.4)

From the definitions of Hubble parameter and deceleration parameter

$H = \frac{\dot{a}}{a}$ and $q = -\frac{a\ddot{a}}{\dot{a}^2}$

Then (2.4) become

$$2H^2 - 2qH^2 = \frac{\rho_m}{\emptyset} - 5H\frac{\dot{\emptyset}}{\emptyset} - \frac{\ddot{\emptyset}}{\emptyset} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (2.5)$$

According to Brans-Dicke theory the gravitational constant $G = \frac{1}{\emptyset}$

And relative rate of change $\frac{\dot{G}}{G} = -\frac{\dot{\emptyset}}{\emptyset}$ ____________________ (2.6)

The values of cosmological parameters at the present time $t = t_0$ used in this article, are given below.

Here t_0= age of the universe.

$H_0 = 2.39 \times 10^{-18} sec^{-1}$, $t_0 = 4.13 \times 10^{17} sec$,

$\rho_{m_0} = 2.97 \times 10^{-27} kg/m^3$, $G_0 = 6.67 \times 10^{11} Nm^2Kg^{-2}$

$\emptyset_0 = \frac{1}{G_0} = 1.50 \times 10^{-10} N^{-1}m^{-2}Kg^2$, $q_0 = -0.55$

3. Empirical Model for the Scalar Field (ϕ)

We assume the following ansatz to represent the time dependence of the scalar field

$$\emptyset = \frac{\emptyset_0}{1+e^{-k(t-t_0)}} , \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (3.1)$$

k is parametric constant with dimension t^{-1} .

$$\frac{\dot{\emptyset}}{\emptyset} = k \text{ and } \frac{\ddot{\emptyset}}{\emptyset} = k^2 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (3.2)$$

Substitute these values in (2.6)

$$k^2 + 5Hk + 2H^2(2-q) - \frac{\rho_m}{\emptyset} = 0 \qquad (3.3)$$

Equation (2.7) is a relation among some time dependent cosmological parameters

This relation must be valid at all instants of cosmic time (including the present time). Replacing these parameters by their values at the present time (i.e., at $t = t_0$)
we get

$$k^2 + 5H_0k + 2{H_0}^2(2-q_0) - \frac{\rho_{m_0}}{\emptyset_0} = 0 \qquad (3.4)$$

Roots of this quadratic equation

$$k = \frac{-5H_0 \pm \sqrt{25{H_0}^2 - 8{H_0}^2(2-q) + 4\frac{\rho_{m_0}}{\emptyset_0}}}{2} \qquad (3.5)$$

Then $k_1 = -3.37 \times 10^{-18}$ and $k_2 = -8.58 \times 10^{-18}$

And $G = \frac{1}{\emptyset_0}\left(1 + e^{-k(t-t_0)}\right)$ (3.6)

From the equations (2.6) and (2.8)

$\frac{\dot{G}}{G} = -k_1 = 3.37 \times 10^{-18} = 1.06 \times 10^{-10}\ Yr^{-1}$ and

$$\frac{\dot{G}}{G} = -k_2 = 8.58 \times 10^{-18} = 2.70 \times 10^{-10}\ Yr^{-1}$$

These values are constant with upper limit $4 \times 10^{-10}\ Yr^{-1}$
For $\left|\frac{\dot{G}}{G}\right|_0$ as stipulated by Steven Weinberg [23]

4. Results and discussion:

Based on equation (3.1), Figure 1 depicts the variation of the scalar field ($\emptyset$) as a function of time for two values of the parameter k obtained from equation (3.4). Because both k values are negative, $\emptyset$ declines over time, as predicted by equation (3.1). Figure 2 depicts the temporal evolution of the gravitational constant (G) for two values of the parameter k obtained from equation (3.6) for two different values of the parameter k . The gravitational constant (G) is found here to be increasing with time due to the negative values of m. This nature of time variation of G is in qualitative agreement with the findings of some recent studies based on models which are completely different from the present one [24-26]. Both $\emptyset$ and G have been plotted here in logarithmic scales for a greater visual clarity of their values obtained from the graphs. Logarithm of ϕ and G vary linearly with time, in accordance with equations (3.1) and (3.6) respectively.

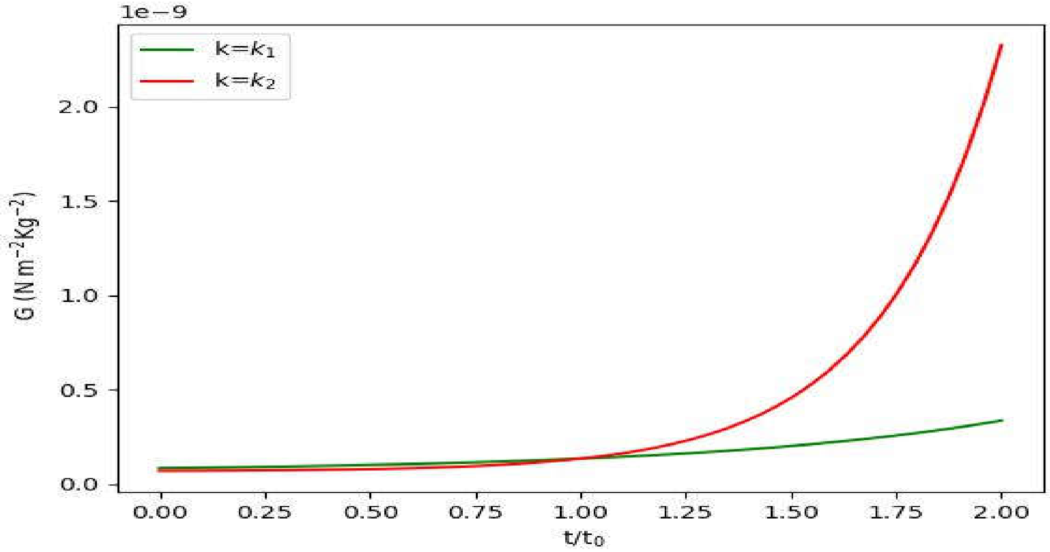

Figure 1 : Plots of Brans-Dicke scalar field Φ versus time for two values of k.

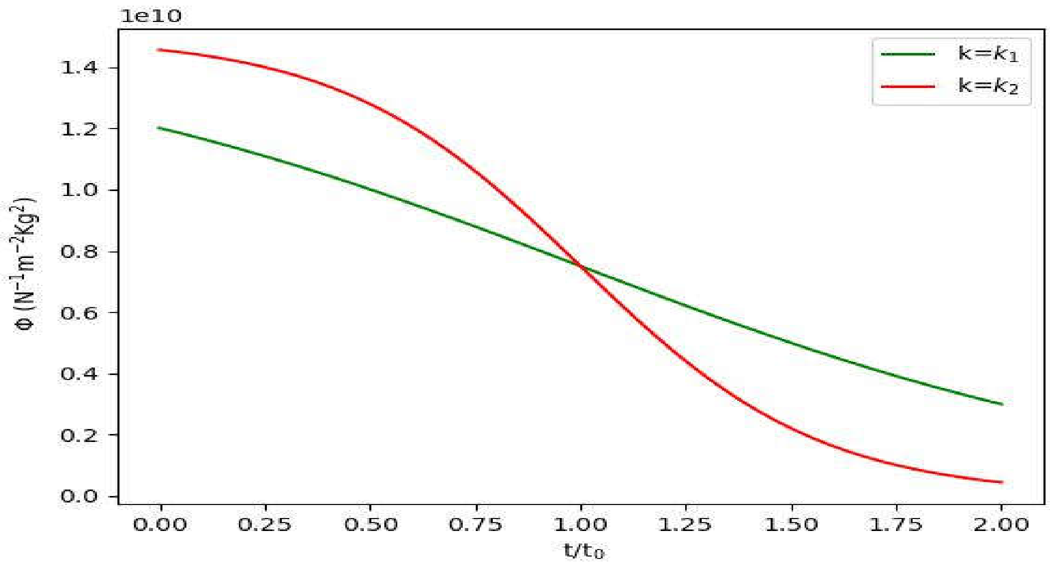

Figure 2 : Plots of Brans-Dicke Gravitational constant Gversus time for two values of k.

1. Conclusion:

It would be fascinating to learn which of the two values of the parameter m produces more accurate findings. Unless we have a large enough volume of very reliable experimental data on the time variation of the gravitational constant (G), both values must be considered equally legitimate. On the basis of equation (3.3), we created equation (3.4), which assumes that the solution of Brans-Dicke field equations leads to the exact values of the cosmological parameters (k, q_0 , ρ , m_0 & ϕ_0)which have been precisely obtained from recent astrophysical data and utilized to determine the values of the parameter k , using equation (3.4). The correctness of this assumption determines the theoretical validity of the entire formulation. The fact that the values of the cosmological parameters obtained by solving the field equations differ from their values (mentioned in section 2) established from recent astrophysical data and employed in the present investigation may explain the failure to identify a unique value for the parameter k.

As an extension of this work, one may replace equation (3.1) with numerous additional ansatzes expressing the temporal variation of the scalar field (ϕ), in order to determine the time dependence of the gravitational constant (G) and compare the results to those of the current study. The current formulation has the advantage of being based on a very simple theoretical concept or plan that has been accomplished here using a very simple mathematical scheme.

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