Advances in Chemical and Spring Balance Weighing Designs: A systematic review

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Abstract

This paper deals with the review of the literature on Chemical and Spring Balance Weighing Designs. The Weighing problems were initially originated by Yates and after that Hotelling, who led to a precise formulation of the Weighing problem. Eventually, with the passage of time, this problem expanded to form a branch of Design of Experiment. The weighing problem was originated by Yates and Hotelling, which is concerned with finding the weights of 'v' objects in 'n' weighing operations. Overtime, attention was also given to the problem of determining optimal designs in terms of the A-, D-, and E- optimality criteria for both Chemical and Spring Balance Weighing Designs. Currently, significant growth is observed in this field.

Keywords: Weighing Designs, Chemical Balance Weighing Designs; Optimum Chemical Balance Weighing Designs; Spring Balance Weighing Designs; Balanced Incomplete Block Designs; Variance Balance Designs; Efficiency Balance Designs.

1. Introduction

"Weighing" is an important process used in science, industry, and daily life, which is used to measure the mass or weight of an object using a balance or scale. Weighing is an accurate process which is used in various fields like Chemistry, Pharmaceuticals, Agriculture, Engineering and Commerce, where precise quantities directly affect results, safety and quality.

Weighing is very important and vital part of our existence. Every human being present on this planet is connected to weight in some way or the other. From the moment we are born and throughout our daily lives, weighing and measuring have been an important and often vital part of our existence. Our bodies, the food we eat and all the products we use as an integral part of modern living have all been weighed and measured at some stage in their development. Weights and measures are undoubtedly one of the greatest and most important inventions of human race, ranking alongside the wheels in the evolution of civilization. Commerce would not have progressed beyond the barter system without the inventions of a system of weights and measures. There are three elements to the weighing story and each evolved over the 6,000 years of its history. First, we have the development and use of weights; then the different weighing machines and apparatus; and finally, the introduction of weights and measures to control commercial transactions.

As technology has evolved, weighing machines have become more automatic and can now connect to computers. In various fields it has become easier to track results, keep records and analyze data in more effective way.

2. History of Chemical and Spring Balance Weighing Designs

Weighing Design is one of the research fields of Design of Experiment. "Sir Roanld A.

Fisher" is known as the father of Design of Experiments. He authored a book on DOE in 1935.

Study of weighing problem originated in a casual illustration furnished by Yates. The precise formulation of such problems can be found in Hotelling. Hotelling and Yates have shown that the individual weights may be determined more accurately by weighing the objects in combinations rather than weighing each one separately. Over the years the problem has attained a distinctive growth, has branched out in different directions, and has meanwhile acquired the status of a problem in the design of experiments. The problem has also become associated with the name of Hadamard and has given noticeable momentum to research in the extension of the Hadamard determinant problem. The experimental designs are applicable to a broad class of problems of measurement of similar objects. The chemical balance problem (in which objects may be placed in either of the two pans of the balance) is almost completely solved by means of designs constructed from Hadamard matrices.

3. Preliminaries

Weighing Designs: Weighing designs consist of n groupings of the p objects and suppose we want to determine the individual weights of p objects. We can fit the results into the general linear model as

$$y_{n\times 1} = X_{n\times p}w_{p\times 1} + e_{n\times 1} \tag{1.1}$$

The elements of matrix X = (x_{ij}), i = 1,2………., n, j = 1,2 ………p, take the value as

$$x_{ij} = \begin{Bmatrix} +1\ if\ the\ jth\ object\ is\ placed\ in\ the\ left\ pan\ in\ the\ ith\ weighing \\ -1\ if\ the\ jth\ object\ is\ placed\ in\ the\ right\ pan\ in\ the\ ith\ weighing \\ 0\ if\ the\ jth\ object\ is\ not\ weighed\ in\ the\ ith\ weighing \end{Bmatrix}$$

where y is an $n \times 1$ random column vector of the observed weights, w is the p×1 column vector representing the unknown weights of objects and e is an n× 1 column vector of errors such that E(e) = 0_n and E(ee′) = $\sigma^2 I_n$,

where 0_n is the $n \times 1$ column vector with zero elements everywhere; in the n× n identity matrix

"E" stands for Expectations and "e" stands for transpose of e.

The normal equations estimating w are of the form

$$X'X\hat{w} = X'y \tag{1.2}$$

where ŵ is the vector of the weights estimated by the least square method.

A Chemical balance weighing design is said to be singular or nonsingular, depending on whether the matrix X′X is singular or nonsingular, respectively. It is obvious that the matrix X′X is nonsingular if and only if the matrix X is of full column rank (=p). Now, if X is of full rank, that is, when X′X is nonsingular, the least square estimate of w is given by ŵ =$(X'X)^{-1}$X′y

and the variance – covariance matrix of ŵ is Var(ŵ) = $\sigma^2(X'X)^{-1}$

Chemical Balance Weighing Designs: When the objects are placed on two pans in a chemical balance, we shall call the weighings two pan weighing and the design is known as two pan design or chemical balance weighing design. In chemical balance the objects can be placed either on one pan or on both pans for each weighing. If in a weighing design, suppose we are given p objects weighed in n weighing operations, the elements of design matrix X = $\{x_{ij}\}$ takes the values as

$$x_{ij} = \begin{Bmatrix} +1 \; if \; the \; jth \; object \; is \; placed \; in \; the \; left \; pan \; in \; the \; ith \; weighing \\ -1 \; if \; the \; jth \; object \; is \; placed \; in \; the \; right \; pan \; in \; the \; ith \; weighing \\ 0 \; if \; the \; jth \; object \; is \; not \; weighed \; in \; the \; ith \; weighing \end{Bmatrix}$$

The n^{th} order matrix X = $\{x_{ij}\}$ is known as the design matrix of the chemical balance. If n weighing operations are to determine the weights of p = n objects, the minimum variance that each estimated weight might have been σ^2/n.

Optimum Chemical Balance Weighing Designs: A non-singular chemical balance weighing design is said to be optimal for the estimating individual weights of objects if the variance of their estimators attains the lower bound given by -

$$\text{Var}(\hat{w}) = \sigma^2/\text{m}, \text{j} = 1, 2, \ldots\ldots\ldots\ldots, \text{p} \tag{1.3}$$

where m = $\{m_1, m_2, \ldots\ldots\ldots, m_p\}$

Spring Balance Weighing Designs: In Spring balance only one pan is available for placing the objects. so, the elements of the design matrix assume only the values 1 and 0 according as the corresponding object is weighted in the combination or not.

The results of n weighing operations the individual weights of p objects fit into the linear model

$$y_{n\times1} = X_{n\times p}w_{p\times1} + e_{n\times1} \tag{1.4}$$

If $X'X$ is non – singular, the least square estimate of β is given by

$$\beta = (X'X)^{-1}X' \tag{1.5}$$

With covariance matrix $\sigma^2(X'X)^{-1}$ If $X'X$ is singular take 't' additional weighings (t > 0) made with all the 'p' objects being placed in the pan.

Balanced Incomplete Block Designs: The Balanced Incomplete Block Designs are arranged in blocks or groups that are smaller than a complete replication in order to eliminate heterogeneity to a greater extent than is possible with randomized block designs and Latin square designs.

An arrangement of v treatments in b blocks of k plots each (k < v) is known as BIBD if

(i) Each treatment occurs once and only once in r blocks and

(ii) Each pair of treatments occurs together in λ blocks. The

integers v, r, b, k and λ are called the parameters of the BIBD.

Variance Balanced Designs: A block design is said to be balanced if every elementary contrast of treatment is estimated with the same variance. In this sense this design is also called a variance balanced design. It is well known that block design is variance balanced if and only if it has

$$C = \mu\, [\, I_v - (1/v)\, 1_v 1_v'] \tag{1.6}$$

where μ is the unique nonzero eigenvalue of the matrix C with the multiplicity (v – 1), $I_v = v \times v$ identity matrix.

Efficiency Balanced Designs: A block design is called efficiency balanced if every contrast of treatment effects is estimated through the design with the same efficiency factor.

Let us consider the matrix M_o given by Calinski (1971)

$$M_o = R^{-1}NK^{-1}N' - (1/n)\ I_v r' \tag{1.7}$$

and since M_o S = μ S

where φ is the unique non zero eigen value of M_o with multiplicity (v – 1) $and\ M_o$ is defined as above. Calinski (1971) showed that for such designs every treatment contrast is estimated with the same efficiency (1 – μ) and N is a EB block design if and only if

$$M_o = \mu\,(I_v - (1/n)\ 1_v r') \tag{1.8}$$

Kageyama (1974) proved that for the EB Block design N, eq (1.8) is fulfilled if and only if

$$C = (1 - \mu)\ (R - (1/n)\ r\,r' \tag{1.9}$$

4. Related Work

Weighing design is a vast branch in which many Statisticians have contributed. The Weighing problem initially originated by Yates and after that Hotelling. Yates and Hotelling understood the concept of Weighing Design very well and concluded that individual weights can be calculated even better if we weigh objects in combination rather than weighing each one separately. Hotellingshowed that the minimum attainable variance for each of the estimated weights for a Chemical Balance Weighing Design is σ^2/n. He showed that variance of each of the estimated weights attains the lower bound if and only if $X'X$ = nI_p. This is the required condition for any design to be called as Optimum Chemical Balance Weighing Design. There is extensive literature on constructing methods for Optimum Chemical Balance Weighing Designs. Mood [46], Dey [36, 37], Banerjee [8, 9, 10, 11], Raghav Rao [47, 48, 49, 50] are some of the prominent authors of this field. Apart from this, we are also going to see a lot of work in Spring Balance Weighing Designs. A lot of work has also been done on the optimality criteria of Spring Balance Weighing Designs.

A good deal of work has been done by Saha [51], Kageyama and Saha [43] as they provided several methods for construction of Optimum Chemical Balance Weighing Design without any restrictions on the number of objects placed on the either pan is available. They gave us the idea of how we can obtain from the incidence matrices of balanced incomplete block designs for p = v objects. Kageyama and Saha [44] used incidence matrices of balanced incomplete block design for p = v + 1 objects in n = 4(r – λ) weighings. In the same case, another method of construction has been proposed by Ceranka and Katulska [13]. They provide us the necessary and sufficient conditions in which a chemical balance weighing design for v + 1 objects was optimal. They constructed the necessary and sufficient conditions for optimum biased spring balance weighing designs and chemical balance weighing designs with non – homogeneity of the variances of errors and also tried to find out the relationship between them.

In the field of Spring balance weighing design, Jacroux et al. [42] proposed a technique for finding *Φ*- optimal designs. They used spring balance for weighing v objects in b weighing. They tried to minimize error and maximize accuracy. For this, they used optimality function and used these methods to find A – optimal, D – optimal and E – optimal.

Ceranka et al. [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31] have been done a lot of work in the field of chemical balance weighing designs. A path to deal with the problem of estimating individual weights of objects, using a chemical balance weighing design under the restriction on the number in which each object is weighed, this concept is provided by Ceranka et al [14]. They also constructed optimum chemical balance weighing designs by using incidence matrix of ternary balanced block design.

By using a Chemical Balance Weighing Design, Ceranka et al. [14] proposed the problem of estimating individual weights of objects under the restriction on the number of times in which each object was weighed. From this design, they also obtained a lower bound for the variance of each of the estimated weights and a necessary and sufficient condition to attain this lower bound was given. They constructed chemical balance weighing designs by using incidence matrices of balanced bipartite block design under the restriction on the number in which each object was weighed.Ceranka et al. [15] under the restriction on multiplicity of each object weighing studied the problem of estimating individual weights of objects using a chemical balance weighing design. The incidence matrices of balanced bipartite block designs for v treatments were used to construct the optimum chemical balance weighing design for p = v + 1 object was proposed.

Ceranka et al. [16] constructed the design matrix X of optimum chemical balance weighing design under the restriction on the number in which each object is weighed by using incidence matrices of balanced incomplete block designs and balanced bipartite block design.

In the field of chemical balance weighing designs, Ceranka et al. [17] aims to find out the solution of problem of estimating individual weights of objects. They also concluded that all variances of estimated weights were equal and they attained the lower bound. They proposed a necessary and sufficient condition to attain this lower bound was given. They used incidence matrices of balanced bipartite block designs and ternary balanced block designs for the construction of the design matrix X of optimum chemical balance weighing designs.

Ceranka et al. [20, 21, 22, 23, 24, 25] discussed another method of construction for optimum chemical balance weighing designs for p = v + 1 objects in which they used the incidence matrices of ternary balanced block designs for v treatments. They provided some conditions for the existence of an optimum chemical balance weighing design for p = v objects lead to the existence of an optimum chemical balance weighing design for p = v + 1 objects and the existence of an optimum chemical balance weighing design for p = v + 1 objects lead to the existence of an optimum chemical balance weighing design for each p < v + 1.

Ceranka et al. [26, 27, 28, 29] problem of estimation of individual weights of objects using a chemical balance weighing design under the restriction on the number of times in which each object was weighted. For this, they assumed that the errors are uncorrelated with different variances. They also provided the necessary and sufficient condition under which the lower bound of variance of each of the estimated weights is attained. They used the incidence matrices of the balanced incomplete block designs and the ternary balanced block designs for the new construction method of optimum chemical balance weighing designs.

For the new construction methods of the optimum chemical balance weighing design for p = v + 1 objects, which was based on the incidence matrices of the balanced incomplete block designs and the balanced bipartite weighing designs for v treatments Cerenka et al. [30] assuming that not all objects in each measurement were included.

Under the assumption that the measurement errors are uncorrelated and they have different variances Ceranka et al. [31] considered the problem of estimation of unknown weights of p = v + 1 objects in the field of the chemical balance weighing design.

Graczyk et al. [39] studied the special type of weighing design called A – optimal spring balance weighing design. He constructed this design by using incidence matrices of balanced incomplete block designs and group divisible designs, assuming that errors are uncorrelated with different variances.

Awad et al. [1, 2, 5] proposed some different construction methods for design matrix X of optimum chemical balance weighing designs which were based on the incidence matrices of symmetric balanced incomplete block designs. They also obtained some pairwise balanced designs which are efficiency as well as variance balanced. In these theoretical aspects, they also proposed some theorems, lemmas and corollaries for better and precise results.

In the direction of Spring Balance Weighing Designs, the problem of estimation of individual weights of objects in E – optimal spring balance weighing design under the assumption that the measurement errors are equal correlated considered by Ceranka et al. [32]. They mainly focused on finding the largest eigenvalue of the inverse of the information matrix.

By using the incidence matrices of the known symmetric balanced incomplete block designs Awad et al. [3] introduced some new methods of construction for optimum chemical balance weighing designs and pairwise efficiency and variance balanced designs. They also provided A – optimality criteria for the newly constructed chemical balance weighing designs, and also A – optimality of previously constructed optimum chemical balance weighing designs.

Ceranka et al. [33] constructed a new method for A – optimal spring balance weighing design which was based on the incidence matrices of balanced incomplete block design. They also provided suitable example for better understanding. By using spring balance weighing designs they estimated unknown weights of p objects. They concluded that if the first column of the design matrix has elements equal to one only, the weighing design is called biased.

Awad et al. [4] constructed some new methods of the optimum chemical balance weighing designs which were based on the incidence matrices of the known balanced incomplete block designs, balanced bipartite block designs and ternary balanced block designs. They also provided conditions under which the new constructed designs become A – optimal.

Another important work has been done by Awad et al. [6]. They constructed equireplicate Variance Balanced (VB) and Efficiency Balanced (EB) design with unequal block sizes. For this construction they used 2^n – symmetrical factorial design by deleting the control treatment and merging all the main effects with highest order interactions separately and after that they checked optimality of the constructed design and found it to be universally optimal.

Awad et al. [7] constructed equireplicate Variance Balanced (VB) and Efficiency Balanced

(EB) design with unequal block sizes. This construction is based on 3^n – symmetrical factorial design by deleting control and some other unimportant treatment combinations and merging some of the treatment combinations. Further they checked the optimality of constructed designs and found it to be universally optimal.

To estimate the weights of the individual objects Bhatra et al. [12] provided the two methods for the construction of spring balance weighing designs which was based on balanced incomplete block designs and Mutual Orthogonal Latin Squares.

Some results of construction of A, D, and Regular A – optimal Spring balance weighing designs was given by Khurana et al. [45]. They used different orthogonal and covering array designs for these constructions. Khurana et al. [45] estimate the weights of individual objects in A, D, and Regular A – optimal spring balance weighing design. They concluded with the bound, depending on whether the number of objects in the experiment are odd or even.

Assuming that the measurements are equally positively correlated and have the same variances Graczyk et al. [40] found out the solution to the problem of determining a chemical balance weighing design to become D – optimal. They provided a method of adding three measurements to a regular D – optimal chemical balance weighing designs to obtain a highly D – efficient chemical balance weighing designs. They used previously constructed methods for further findings.

In the direction of Spring balance weighing designs Graczyk et al. [41] discussed some problems relating to efficient spring balance weighing designs. They provided an example to explain how such designs can be constructed, and also described a condition that shows connection of the different parameters of these designs.

5. Conclusion

This review paper emphasizes the crucial role of Chemical and Spring balanced weighing designs in addressing challenges associated with different construction methods. Weighing designs are useful in various fields like pharmaceuticals, industries, chemical laboratories, manufacturing and industrial processing units. Weighing designs are utilized to enhance the precision of the estimates. Integration of modern methodologies with digital techniques is employed to obtain more meaningful results. Weighing Designs continue to provide a robust foundation for both theoretical exploration and practical applications.

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