Perspectives and Insights in the Field of Resolvable and Nearly Resolvable Block Designs: A Review

Awad Rashmi [1], Agrawal Bharti [2] and Yadav Shalini [3]*
[1]Assistant Professor, School of Statistics, D.A.V.V., Indore, M.P., India
[2]Assistant Professor, Department of Mathematics, Medicaps University, Indore, M.P., India
[3]Research Scholar, School of Statistics, D.A.V.V., Indore, M.P., India

Abstract
Resolvable and nearly resolvable designs are important combinatorial structures that play a vital role in experimental designs and statistical analysis. This review paper presents a comprehensive study of the theoretical foundations, construction techniques and major developments in resolvable and nearly resolvable block designs. The main attention is given to their connections with variance balanced and efficiency balanced designs, with an evaluation of methodologies used for their construction. By reviewing and structuring the existing literature, the aim of this paper is to provide a clear understanding about resolvable and nearly resolvable block designs. This review also highlights the potential directions for future research.

Keywords: Balanced Incomplete Block Designs; Symmetric Balanced Incomplete Block Designs; Variance Balanced Designs; Efficiency Balanced Designs; Resolvable Designs; Affine Resolvable Designs; α – Resolvable Designs; Affine α-Resolvable Designs; Nearly Resolvable Designs; Nearly μ- Resolvable Designs; Optimality.

1. Introduction

Design of experiments is a statistical method used to plan, conduct, and analyse experiments in order to understand the relationships between variables and how they affect a response. It is a crucial tool for researchers in a variety of fields, including engineering, manufacturing, biology, and psychology.

There are three basic principles of a design which were developed by Sir Ronald A. Fisher; randomization (to remove bias), replication (to estimate experimental error), blocking (to reduce variability).

Sir Ronald A. Fisher (1890- 1962), the father of statistics has the credit of developing the concept of statistical control and the use of randomization in experiments. His work laid the foundation for modern design of experiment techniques, such as analysis of variance (ANOVA) and the development of statistical models.

Combinatorial design theory, a very wide area of discrete mathematics, deals with the arrangement of elements into specified patterns. It provides mathematical structures, such as balanced incomplete block designs (BIBD) and latin squares, to ensure that treatments are compared fairly, minimize the number of required experiments, and enhance the accuracy of statistical analyses in fields like agriculture, biology, and clinical trials. Among this wide range of combinatorial structures, block designs have a special place due to their theoretical and practical uses. A block design provides a systematic way of organizing treatments in experimental units. The commonly used block designs are randomized (complete) block designs, balanced incomplete block designs (BIBD), resolvable designs, partially balanced incomplete block designs (PBIBD) etc. Among these, balanced incomplete block designs (BIBDs) are most widely studied. A block design is said to be resolvable, if its blocks can be divided into replicates such that each treatment appears once in every replicate. To achieve orthogonality between treatments and nuisance factors and making experiments more reliable, resolvability plays important role. It is useful in sequential experiments, multi-site studies and experiments in which there are multiple individuals. The most common application of resolvable designs is in agricultural field trials. When resolvable designs are not feasible then a practical alternative emerges which is known as nearly resolvable designs. The perception of resolvability in combinatorial designs was introduced by Bose in 1942, however it was extended in the context of experimental design with the development of nearly resolvable designs by Shrikhande and Raghavarao in 1960s and more recently by Yadav, Dash and Singh [69].

2. History

Design of experiments was developed by Sir Ronald A. Fisher in the 1920s and 1930s at Rothamsted Experimental Station, an agricultural research station. In the period 1936-1940, while working on latin square designs, Yates introduced the idea of resolvability. The concept of resolvability in block designs can be seen in Kirkman's school girl problem given in 1850s. Later, in the year 1942 the concept of resolvable designs was introduced by R.C. Bose. Resolvable designs are highly useful as they facilitate effective inter-block and intra-block data analysis, further improving efficiency and precision of the experiment. Whereas, in Nearly Resolvable Designs which are a generalization of resolvability, blocks can be divided into sets, but some treatments do not appear in every set. The idea of nearly resolvable designs for incomplete block designs was given by Shrikhande and Raghavarao in 1960s.

3. Preliminaries

Balanced Incomplete Block Designs (BIBD): An arrangement of v treatments in b blocks of k plots each ($k<v$) is known as BIBD, if (i) Each treatment occurs once and only once in r blocks and (ii) Each pair of treatments occurs together in λ blocks.

Necessary conditions for the existence of the B.I.B.D. are:

(i) $vr = bk$, (ii) $\lambda (v – 1) = r (k-1)$, (iii) $b \geq v$ (Fisher's Inequality)

Symmetric Balanced Incomplete Block Designs: A BIBD is said to be symmetric if $b = v$ and $r = k$. In this case incidence matrix N is a square matrix i.e. $N' = N$. In case of symmetric balanced incomplete block designs any two blocks have λ treatments in common.

Variance Balanced Designs: A block design is said to be variance balanced if it allows the estimation of all normalized treatment contrasts with the same variance.
Let us consider the matrix $C = R - NK^{-1} N'$
where $R = \text{diag}(r_1, r_2,\ldots\ldots r_v)$, $K = \text{diag}(k_1, k_2,\ldots\ldots k_b)$
Kageyama [41] established that N is variance balanced block design if and only if

$$C = \eta\,(I_v - \frac{1}{v} 1_v\, 1'_v)$$

where η is the unique non zero eigen value of C with multiplicity $v – 1$; I_v is the unit matrix of order v; I_v is v x 1 vector all of whose elements are one.

Efficiency Balanced Designs: A block design is said to be efficiency balanced if every contrast of treatment effects is estimated through the design with the same efficiency factor.

Let us consider the matrix M_0 given by Calinski [21]

$$M_o = R^{-1} NK^{-1} N' - \frac{1}{n} 1_v\, r' \tag{1.1}$$

$$M_o S = \mu S$$

where $T= [\ T_1\ T_2\ldots\ldots\ldots T_v]'$ is the vector of treatment totals, T_i is the total yield for the i^{th} treatment, μ is the unique non zero eigen value of M_o with multiplicity $(v – 1)$ and M_o is given as (1.1).

Calinski [21] showed that for such designs every treatment contrast is estimated with the same efficiency (1- μ) and N is an efficiency balanced design if and only if

$$M_o = \mu \left(I_v - \frac{1}{n} 1_v r' \right) \qquad (1.2)$$

Kageyama proved that for the efficiency balanced block design N, equation (1.2) is fulfilled if and only if $C = (1 - \mu)(R - \frac{1}{n} rr')$

Partially Balanced Incomplete Block Designs: Given an association scheme with m classes ($m \geq 2$), a Partially Balanced Incomplete Block (PBIB) design can be constructed based on this scheme if the ν treatments can be arranged into b blocks such that:

1. Each block contains k (< ν) distinct treatments.
2. Each treatment occurs in exactly r different blocks.
3. If two treatments α and β are mutually i-th associates in the association scheme, then α and β occur together in exactly λ_i blocks. The integer λ_i does not depends on the pair (α, β) being i-th associates, where i = 1, 2, ……., m. Moreover, not all λ_i's are necessarily equal.

Group Divisible Designs: A two-associate PBIB design is called a Group Divisible (GD) design if there are v = mn treatments, which can be divided into m groups of n treatments each, such that:

- Any two treatments within the same group are first associates.
- Any two treatments belonging to different groups are second associates.

Here m, n ≥ 2, $n_1 = (n - 1)$ and $n_2 = n(m - 1)$. where n_1 is the number of first associates and n_2 is the number of second associates for each treatment. These group divisible designs are further classified into Singular, Semi Singular and Regular designs.

Resolvable Balanced Incomplete Block Designs: A balanced incomplete block design with v, b, r, k and λ is said to be resolvable if the b blocks can be divided into r groups or sets of b/r blocks each, b/r being an integer, such that b/r blocks forming any of these sets give a complete replication of all the v treatments. The parametric relations $b \geq v + r - 1$ serve as a necessary condition for the existence of a resolvable BIBD.

Affine Resolvable Designs: A resolvable design is said to be affine resolvable if $b = v + r - 1$ and any two blocks from different sets have k^2/v treatments common where k^2/v is an integer.

α-Resolvable Designs: An incomplete block design with the parameter v, b, r, k is said to be α-resolvable if the b blocks of size k each can be grouped into t sets (called α-resolution sets) of β blocks each, such that in each α-resolution set every treatment (or point) is replicated α times and we have, $v\alpha = k\beta$, $b = t\beta$, $r = t\alpha$

Affine α-Resolvable Designs: An α-resolvable block design is said to be affine α-resolvable if every two distinct blocks from the same α-resolution set intersect in the same number, say, q_1, of treatments, whereas every two blocks belonging to different α-resolution sets intersect in the same number, say, q_2, of treatments. Necessary and sufficient condition for an α- resolvable BIB design to be affine α-resolvable with the block intersection number q_1 and q_2 is, $q_1 = k(\alpha - 1)/(\beta - 1)$ and $q_2 = k\alpha/\beta = k^2/v$.

Nearly Resolvable Designs: Abel [66] introduced the concept of nearly resolvable balanced incomplete block designs. A nearly resolvable design, denoted by NRB, is a block design, in which the blocks can be partitioned into partial resolution classes missing single treatment and every treatment of the design is absent from exactly one class; considering all the blocks together it forms a BIBD design.

Nearly μ-resolvable Designs: A design is (nearly) μ-resolvable if it has (nearly) a resolution such that any (v-1) treatments occur μ- times in each of the resolution sets. When μ=1, it leads to a nearly resolvable design in the sense of Abel and Furino.

Optimality: Optimal designs are a class of experimental designs that are optimal with respect to some statistical criterion.
(i) **A-optimality (trace):** A-optimality minimizes the trace of the inverse of the information matrix. This criterion results in minimizing the average variance of the estimates the regression coefficients.
(ii) **D-optimality (determinant):** A popular criterion, D-optimality, seeks to minimize $|(X'X)^{-1}|$ or equivalently maximize the determinant of the information matrix $X'X$ of the design. This criterion results in maximizing the differential Shannon information content of the parameter estimates.
(iii) **E-optimality (eigen value):** E-optimality maximizes the minimum eigen value of the information matrix.

4. Related Work

In 1850, Kirkman introduced the well known 'school girl problem'. The challenge was to arrange three girls in five rows each, across seven days. The arrangement had to be such that every possible pair of girls walks together in the same row on exactly one day. More generally, the problem was to arrange (6n + 3) girls into (2n +1) rows of three, over (3n + 1) days, in such a way that each pair of girls appears in the same row exactly once. In modern terminology, such arrangements correspond to the construction of resolvable balanced incomplete block designs with block size 3.

However, the term "resolvable design" was first introduced by Bose in 1942. He established the conditions needed for a Balanced Incomplete Block Designs (BIBD) to be resolvable or affine resolvable. Yate's [71] work was later expanded by Harshbarger [34] who extended it to rectangular lattices. Bose and Nair [19] applied it to two-replicate designs, also created a special type of resolvable BIBD with two replications, using sample squares, rectangular lattices, or their extensions. These kinds of BIBDs gained attention in both combinatorics and statistics. Later, Shrikhande and Raghavarao [64] expanded Bose's ideas to define α-resolvability and affine α-resolvability. They identified the key conditions for a design to meet these criteria and applied the concepts to certain BIB designs and PBIB designs. Many researchers have contributed to this field by studying the construction and properties of affine α-resolvable BIB designs and PBIB designs. Some of the key contributors include Bailey [12], Banerjee and Kageyama [14], Bose [18], Calinski and Kageyama [22], Clatworthy and Shrikhande [23], Monod and Morgan [39], Kageyama and Mohan [51], Shrikhande and Raghavarao [63]. David [27] worked on cyclic designs. Enormously significant work has been done by statisticians in the area of resolvable designs. Chaudhuri and Wilson [59] provided necessary and sufficient conditions for constructing resolvable balanced incomplete block designs when the block size is 3.

The concept of μ- resolvable balanced incomplete block designs was introduced by S. Kageyama in 1973 with parameters v, $b = m\,t$, $r = \mu t$, k and λ, Kageyama established the inequality $b \geq v+t-1$. The main objective of this note is to improve this bound to $b \geq \max\left(v+t-1, \frac{m^2\lambda+m}{\mu^2}\right)$. Furthermore, this inequality is redefined for μ-resolvable BIBD that are not affine μ-resolvable.

In 1973, Clatworthy [25] provided information about Group Divisible Designs and other Partially Balanced Incomplete Block (PBIB) designs with two-associate classes. Cerenka [24] provided different methods of constructing PBIBDs. Kageyama [51] extended the concept of μ-resolvability to $(\mu_1, \mu_2, \ldots, \mu_t)$-resolvability. Agrawal and Boob [8] developed a new way to construct Affine Resolvable Balanced Incomplete Block Designs using an incidence matrix. Several researchers introduced new methods to construct Resolvable Incomplete Block

Designs. Patterson and Williams [56], Jarrett and Hall [37] and Agrawal and Boob [8] proposed constructions for any number of treatments and block sizes divisible by 3. These are helpful in situations where traditional methods don't work, especially when a complete set of incomplete block designs is difficult to build. Two main methods are often used: (i) Patterson and William's method, which is direct and simple. (ii) Jarrett and Hall's method, which needs an initial block set but becomes easier once that is ready.

Shah [61] also provided conditions under which an α-resolvable incomplete block design is affine α-resolvable. Mukerjee and Kageyama [54] developed the idea of resolvable and affine resolvable variance-balanced designs, explaining the concept of affine (μ_1, μ_2,…, μ_t)-resolvability. They also examined the connection between affine resolvability and variance-balance, the relationship defined by the equation b = v + t - 1.

A new method to construct self-complementary semi-regular group divisible (SRGD) designs using self-complementary balanced incomplete block designs was proposed by Kageyama et al. in [43]. Some existent results of resolvable and α-resolvable group divisible designs are given in literature like Ahmed and Alan [9], Gennian [30, 31].

Banerjee and Kageyama [14] presented the existence of α-resolvable nested incomplete block designs and provided a method to construct such designs from affine α′-resolvable BIB (PBIB) designs for specific values of α and α′.

After the introduction of nearly resolvable designs in design of experiments in 1994, Steven Furino [29] provided the necessary conditions for the existence of nearly resolvable designs. He discussed the required conditions for the existence of NRB (v, k) when v = 1 (mod k) and v = k – 1. He also proved that these conditions are sufficient for k = 2,3,4 and almost sufficient for k = 5, 6. These new methods helped in creating previously unknown designs such as NRB (v, 5) and NRB (v, 6).

Bailey, Monod, and Morgan [12] presented strong optimality criterion for affine resolvable designs originally introduced by Bose. Ionin, Mohan, and Shrikhande [36] extended the idea of resolvability and affine resolvability to pairwise balanced designs by analysing the vector space of linear polynomials in variables corresponding the blocks. They also established the necessary and sufficient conditions for such designs.

Later Tran Van Trung [68] introduced a classical method using inverse planes of even order and developed a recursive construction technique for BIBDs using resolvable BIBDs. He also showed that the approach of Shrikhande and Raghavarao [63] can be extended using near resolvable designs.

Noga Alon and Raphael Yuster [11] studied H-decompositions of complete graphs (Kn) and showed that each has a nearly resolvable version, where H is a fixed graph. In an H-decomposition, the edges of Kn are coloured so that each colour class forms a copy of H (called a member). The resolution number χ(L) is the smallest number of subsets required to partition the decomposition such that no two members in a subset share vertices. They proved that whenever Kn has an H-decomposition, it also has a decomposition L such that $\chi^2(L) \geq$ average degree of H.

Sunanda Bagchi [13] published a paper in which she proposed a general method of constructing group divisible designs and rectangular designs by using resolvable and almost resolvable balanced incomplete block designs. Group divisible designs with $\lambda_2 = \lambda_1 + 1$, Rectangular Designs consisting of two rows and satisfying $\lambda_3 = \lambda_2 = \lambda_1 + 1$ were derived, many of the group divisible designs are shown to be optimal among binary designs concerning all type 1 criteria.

Harri H. and Petteri K. [33] established a connection between nearly resolvable 2-(v, k, λ) designs and a certain class of codes. They specifically studied the nearly resolvable 2-(13, 4, 3) designs and classified them using an orderly algorithm. The thirteen-player whist tournaments were also analyzed based on these designs.

Rudra et al. [60] proposed construction methods for 3-resolvable nested 3-designs and nested 3-wise balanced designs based on affine resolvable 3-designs. Malcolm Greig and Haanpaa [63] established that a near resolvable 2-(2k +1, k, k – 1) design exists if and only if a conference matrix of order 2k + 2 does. A known result on conference matrices allows us to conclude that a near resolvable 2-(2k + 1, k, k − 1) design with even k can only exist if 2k + 1 is the sum of two squares. In particular, neither a near resolvable 2- (21, 10, 9) design nor a near resolvable 2-(33, 16, 15) design exists. For $k \leq 14$, Rudra et al. enumerated the near resolvable 2- (2k + 1, k, k − 1) designs and the corresponding conference matrices.

Morgan J.P. and Brian H. Reck [39] worked on resolvable designs with large blocks. They illustrated the sufficient conditions to achieve optimality under some strong criteria and offered a detailed study of E-optimality with a characterization of E-optimal class. This study also explores resolvable designs with two blocks per replicate in terms of their optimality. The concept of α-resolvability in block designs was established by Kadowaki and Kageyama [41]. The concept had previously been examined in different studies in 1942 for $\alpha = 1$ and precisely for $\alpha \geq 2$ in 1963. In an α- resolvable balanced incomplete block design, three different lower limits for the number of blocks $b \geq v - 1 + t$, $b \geq v - 1 + 2t$, $b \geq 2(v - 1) + t$ will be discussed under certain conditions. Calinski and Kageyama [23] contributed by analysing experimental designs involving affine resolvability. Sinha [67] developed a method to construct optimal group divisible designs from affine resolvable BIB designs.

From 1974–2009 there has been a lot of progress in experimental designs, especially in the area of resolvable designs, with major contributions from Kageyama [42-52]. Kageyama [42] highlighted that affine α-resolvable group divisible designs of regular type does not exist when $\alpha \geq 1$, meaning this concept doesn't apply to regular group divisible designs. Kageyama [47] also showed that affine α-resolvable triangular designs do not exist for $\alpha = 1, 2$, and only partial existence is possible when $\alpha \geq 3$. He later proved that such designs do not exist for α between 1 and 10. Dean Cronvik [26] presented methods for constructing block designs using resolvable designs.

Luis B. Morales [53] formulated the problem of constructing 1-rotational near resolvable difference families as a combinatorial optimization problem where a global optimum corresponds to a desired difference family.

Kadowaki and Kageyama [42] explored ways to build affine α-resolvable PBIB designs (including SGD, SRGD, and Latin square designs) using tools like, Difference Schemes, Hadamard Matrices, BIB designs with $v \leq 100$ and $r, k \leq 20$. Their study revealed that number theory can help in understanding the combinatorial structure of such designs. Later in 2012, Kadowaki and Kageyama [43] proposed new construction methods for affine resolvable SRGD designs. A new method for building SRGD and resolvable SRGD designs was developed using self-complementary balanced incomplete block designs, as proposed by Singh [76]. He also provided a series of resolvable SRGD designs. Shangdi and Huihui [71] proposed methods for building key distribution patterns based on resolvable designs. They explored the connection between resolvable designs and key distribution patterns, and how key distribution patterns can be converted into resolvable designs.

Agrawal et al. [3] developed 3-resolvable regular group divisible designs using incidence matrices derived from known affine resolvable balanced incomplete block designs (BIBDs), and also proposed BIBDs with unequal block sizes. In a subsequent study in the same year, they investigated the optimality of variance and efficiency-balanced affine resolvable designs and resolvable designs with unequal block sizes, establishing that both are universally optimal. They further introduced a construction method for variance and efficiency-balanced (α_1, α_2, ……., α_t) resolvable BIBDs with unequal block sizes based on 2^n -symmetrical factorial designs, focusing on binary block designs and covering cases where $r \leq 30$.

Later Agrawal et al. [5] proposed construction methods for affine resolvable BIBD and affine resolvable rectangular-type PBIBD with unequal block sizes. These constructions are based on the incidence matrices of known resolvable balanced incomplete block designs. Earlier, Kageyama demonstrated that such designs are variance balanced, while Agrawal et al. further established that they are efficiency balanced as well.

Julian R. and Abel R. [40] outlined $v \equiv 1(\text{mod } k)$ as the necessary condition for existence of (v, k, k-1) near resolvable BIBD. A Few more results are established for the existence of (9, 8) frames of type 9t and showed that these exist for all $t \geq 9$.

Bassalygo L.A. and Zinoview V.A. [16] discussed the relationship between balanced incomplete block designs (BIBD), near resolvable balanced incomplete block designs and q-array constant- weight codes. They proved that any balanced incomplete block design B (v, k, 1) generates a near resolvable balanced incomplete block design. Further, they extended this work by proposing that any resolvable BIBD with $\lambda = 1$ results in an optimal equidistant code C_1 with parameters related to design. They also identified additional configurations, such as an optimal equidistant constant composition code C_2 and connected these to near resolvable BIBDs with adjusted parameters.

Durakovic B. [17] explored the historical aspects of Design of Experiments and provided its area of application, revealing that in past 20 years application of Design of Experiments has grown rapidly in manufacturing as well as non-manufacturing domain.

Agrawal et al. [5,7] proposed a method of constructing variance and efficiency balanced (α_1, α_2, …………α_{n-1}) resolvable balanced incomplete block design with unequal block sizes using 2^n - symmetrical factorial design. A detailed investigation confirmed universal optimality of the design.

Banerjee et al. [15] proposed methods of constructions of nearly μ-resolvable designs. They dealt with general construction methods of nearly μ-resolvable balanced incomplete block designs with illustration using one resolvable balanced incomplete block design and some known group divisible designs.

Agrawal et al. [1] suggested a construction technique of affine (μ_1, μ_2, …… μ_t) resolvable balanced incomplete block designs with illustrations. The proposed method is based on the incidence matrix of known affine resolvable balanced incomplete block designs.

Gopinath et al. [32] proposed two new methods of construction to obtain resolvable incomplete block designs for symmetrical and asymmetrical factorial experiments. The designs so generated have orthogonal factorial structure and the main effects are estimated with full efficiency and balance. They presented a catalogue of designs obtainable by this method with number of levels of any factor ≤ 12, along with their efficiency factors.

Shyam S. [65] extended the idea of near resolvability for balanced incomplete block design to partially balanced designs and some such designs are also constructed. Some series of group divisible designs, near resolvable designs and cyclic designs are obtained from α-resolvable group divisible designs, near resolvable triangular and near resolvable cyclic designs, proving these designs are important classes of partially balanced incomplete block designs.

Jha A. and Varghese C. [38] proposed a new series of four- associate partially balanced incomplete block designs in two replications, the block sizes being different. Yadav et al. [69] developed a construction method of α-resolvable BIB designs for even number of treatments and also developed a general construction method of nearly α-resolvable BIB designs. The methods designed by them offer a simplified approach for constructing these designs, addressing some of the complexities and limitations associated with existing methodologies in the field.

5. Conclusion

Resolvable and nearly resolvable designs hold significant importance in designing of combinatorial and experimental theory. This review paper explores the core concept, constructions and wide applications of resolvable and nearly resolvable designs. Recent progress has refined their designs and enhanced applications, still there remains substantial scope for innovation, particularly in developing new construction techniques, extending existing results to broader classes of designs, and exploring deeper connections with related mathematical structures. The development of these designs in diverse domains such as experimental design, network design, and error correcting codes underscore their theoretical and practical importance.

References

1. Agrawal, B. and Awad, R. (2019). Construction of Affine (μ_1, μ_2, ….., μ_t) Resolvable BIB Designs. Shodhkriti, Proceeding of National Conference, 229-233.
2. Agrawal, B., Awad, R. and Banerjee, S. (2019). Construction of α- Resolvable 2- associates PBIB Designs. International journal of Appied Social sciences, Vol 6 (1), 166-170.
3. Agrawal, B. and Banerjee, S., (2016). Some Constructions of 3- Resolvable 2-Associates PBIB Designs. Elixir Statistics 93C, 39807-39813.
4. Agrawal, B. and Banerjee, S., (2016). Some Results on Universal Optimality of Resolvable Designs with Unequal Block Sizes. Elixir Statistics 100, 43360-43370.
5. Agrawal, B., Banerjee, S. and Awad, R., (2016). Some Constructions of Affine Resolvable Designs with Unequal Block Sizes. International journal of Engineering Innovation and Research, Volume 5, Issue 1, 97-102.
6. Agrawal, B., Banerjee, S. and Awad, R., (2018). Some Constructions of α- Resolvable Balanced Incomplete Block Designs. Statistics and Applications, Vol 16 No. 2, pp 65-76.
7. Agrawal, B., Banerjee, S. and Awad, R., (2018). Construction of Resolvable Designs with unequal block sizes using 2^n- Factorial Designs. International journal of innovation in engineering research and management, Volume 5, Issue 1, 1-8.
8. Agrawal, H. L., and Boob, B. S. (1976). A note on method of construction of Affine Resolvable Balanced Incomplete Block design. The annals of Statistics, Vol. 4, No. 5, 954-955.
9. Ahmed, M. A. and Alan, H. (1989). Resolvable group divisible designs with block size three. Discrete Mathematics. North Holland, 77, 5-20.
10. Aldiabat, R. I., Ibrahim, H. and Karim, S., (2019). On the existence of a cyclic near-resolvable (6n + 4) – Cycle System of 2k[2n+9], Journal of Mathematics, 1-10.
11. Alon, N., & Yuster, R. (2000). Every H decomposition of K_n has a Nearly Resolvable Alternative. European Journal of Combinatorics, 21(7), 839-845.
12. Bailey, R. A., Monod, H. and Morgan, J. P. (1995). Construction and optimality of affine-resolvable designs. Biometrika 82, 187–200.
13. Bagchi S., (2004). Construction of Group Divisible Designs and Rectangular Designs from Resolvable and Almost Resolvable Balanced Incomplete Block Desings. Journal of Statistical Planning and Inference. 119(2), 401- 410.

14. Banerjee, S. and Kageyama, S. (1990). Existence of α-resolvable nested incomplete block designs. Utilitas Math. 38, 237–243.
15. Banerjee, S., Awad, R., Agrawal, B. (2018). Some Constructions of Nearly μ-Resolvable Designs, American journal of Applied Mathematical and Statistics, Vol. 6, No. 2, 36-43.
16. Bassalygo, L. A., Zinoviev, V. A. & Lebedev (2018). On m-Near-Resolvable Block Designs and q-array Constant-Weight Codes. Problemy Peredachi Infor-matsii, 54-61.
17. Benjamin D. (2018), Design of Experiments Application, Concepts, Examples: State of Art. Periodicals of Engineering and Natural Sciences Vol 5, No. 3, 421-439.
18. Bose, R. C. (1942). A note on the resolvability of balanced incomplete block designs. Sankhya_ A 6, 105–110.
19. Bose, R. C. and Nair, K. R. (l962). Resolvable incomplete block designs with two replications. Sankhya 24, 9-24.
20. Bose, R. C., and Nair, K. R. (1939). Partially balanced incomplete block designs. Sankhya, 4, 337-372.
21. Bose, R. C., Clatworthy, W. H. and Shrikhande, S. S. (l954). Tables of partially balanced designs with two-associate classes. N. C. Agr. Expt. Sta. Tech. Bull. l07.
22. Caliński, T. and Kageyama, S. (2000). Block Designs: A Randomization Approach, Vol. I: Analysis, Lecture Notes in Statistics 150, Springer, New York.
23. Caliński, T. and Kageyama, S. (2008). On the analysis of experiments in affine resolvable designs, J. Statist. Plann. Inference 138, 3350–3356.
24. Ceranka, B. (1975) Affine resolvable balanced incomplete design. Zastosowania Matematyki Applications Mathematica XIV, 4.
25. Clatworthy, W. H. (1973). Tables of Two-Associate-Class Partially Balanced Designs. NBS Applied Mathematics Series 63, U.S. Department of Commerce, National Bureau of Standards, Washington, D.C.
26. Crnkovic, D. (2009). Some block designs constructed from resolvable designs. Math. Maced. Vol.7, 39-44.
27. David, H. A. (1967). Resolvable cyclic designs. Sankhya A 29,191-8.
28. Ehrenfeld, E. (1955). On the efficiency of experimental designs. Annals of Mathematical Statistics, 26, 247-255.
29. Furino, S. (1995). Existence results for near resolvable designs. Journal of combinatorial designs, 3(2), 101-113.
30. Gennian, G. (2002). Resolvable group divisible designs with block size four. Discrete Mathematics .243, 109-119.
31. Gennian, G. (2006). Resolvable group divisible designs with block size four and index three. Discrete Mathematics 306, 52-65.
32. Gopinath, P. Prasad, R. and Mandal, B.N. (2021). Construction of resolvable incomplete block designs for estimating main effects with full efficiency. Published online, pages 1923-1936.
33. Haanpaa, H., and Kaski, P. (2005). The Near Resolvable 2-(13, 4, 3) designs and thirteen player whist tournaments. Designs, Codes and Cryptography, 271-285.
34. Harshbarger, B. (1946). Preliminary report on the rectangular lattices. Biometrics 2 115–119.
35. Harshbarger, Bs. (1947). Rectangular lattices. Va. Agr. Expt. Sta. Mem. 1, 1-26.
36. Ionin Y. J., Mohan and Shrikhande S. (1998). Resolvable pairwise balanced designs. Journal of Statistical planning and inference, 72, 393-405.
37. Jarrett, R. G. and Hall, w. B. (1978). Generalized cyclic incomplete block designs. Biometrika 65(2), 397-401.

Jha, A. and Varghese, C. (2024). A new series of Affine Resolvable PBIB (4) Designs in two replicates. Ars Combinatoria 161(1): 89-94.

39. J.P. Morgan and Brian, H. Reck. (2007). Resolvable Designs with Large Blocks. The annals of Statistics, Vol 35, No. 2, 747-771.
40. Julian R., Abel R., (2017). Existence of near resolvable (v, k, k − 1) BIBDs with k ∈ {9,12, 16} Australasian journal of Combinatorics, 67(1), 25–45.
41. Kadowaki, S. and Kageyama, S. (2008). On bounds of the number of blocks in α-resolvable BIB designs. Hiroshima University, Higashi-Hiroshima 739-8524.
42. Kageyama, S., and Kadowaki, S. (2009). Existence of affine α- resolvable PBIB designs with some constructions. Hiroshima Math. J. 39, 293-326.
43. Kadowaki, S. and Kageyama, S. (2012). New construction methods of Affine Resolvable SRGD designs. Journal of Statistical Theory and practice, 6, 129-138.
44. Kageyama, S. (1973). On μ-resolvable and affine μ-resolvable balanced incomplete block designs. Ann. Statist. 1, 195–203.
45. Kageyama, S. (1974). On properties of efficiency balanced designs. Commun. Statist. – Theor Math., A9, 597-616.
46. Kageyama, S. (1976). Resolvability of block designs. Ann. Statist. 4, 655–661.
Kageyama, S. (1977). Conditions for α-resolvability and affine α-resolvability of incomplete block designs. J. Japan Statist. Soc. 7, 19–25.
48. Kageyama, S. (2007). On non-existence of affine α-resolvable triangular designs. J. Statist. Theor. Practice 1, 291–298.
49. Kageyama, S. (2008). Non-validity of affine α-resolvability in regular group divisible designs. J.Statist. Plann. Inference 138, 3295–3298.
50. Kageyama S., Banerjee S. and Verma A. (1989). A construction of self complementary semi-regular group divisible designs. Sankhya- B 51, 335–338.
51. Kageyama, S. and Mohan, R. N. (1983) on μ- Resolvable BIB Designs. Discrete Mathematics 45 113-121.
52. Kageyama, S. and Mohan, R. N. (1985). Construction of α-resolvable PBIB designs. Calcutta Statist. Assoc. Bull. 34, 221–224.
53. Luis, B. Morales (2009). Constructing 1-rotational NRDFs through an optimization approach: new (46,9,8), (51,10,9) and (55,9,8) NRBDs, journal of Statistical planning and inference, 139(1), 62-68.
54. Mukerjee, R. and Kageyama, S. (1985). On resolvable and affine resolvable variance-balanced designs. Biometrika, 72, 1, 165-72.
55. Malcolm, G., Harri, H., Petteri, K., (2006). On the coexistence of conference matrices and near resolvable 2-(2k+1,k,k-1) designs, Journal of Combinatorial Theory, Series A, 113(4), 703-711.
56. Patterson, H. D. and Williams, E. R. (1976). A new class of resolvable incomplete block designs. Biometrika 63(1), 83-92.
57. R.J.R Abel, (1994). Forty- three balanced incomplete block designs. Journal of Combinatorial Theory, Series A, 65(2),252-267.
58. Raghavarao, D. (1988). Constructions and Combinatorial Problems in Design of Experiments. Dover, New York.
59. Ray, Choudhari, D. K. and Wilson, R. M. (1971). Solution of Kirkman's School girl problem. Proc. Symp. In pure Mathematics 19 (Am. Math. Soc., Providence, R. I.) 187-203.
60. Rudra, S., Banerjee, S. and Kageyama, S. (2005). Construction of 3- resolvable nested 3-wise designs and 3-wise balanced designs. Australasian journal of combinatorics, Vol. 33, 77-86.
61. Shah, S. M. (1978). On α – resolvability and Affine α–resolvability of Incomplete Block Designs. Annals of Statistics, Vol. 1, No. 2, 468-471.

62. Shangdi, C. and Huihui, W. (2014). Construction for Key Distribution pattern using resolvable designs. Wesa transactions on Mathematics, Vol. 13, 59-68.
63. Shrikhande, S. S. and Raghavarao, D. (1963). A method of construction of incomplete block designs. Sankhya_ A 25, 399-402.
64. Shrikhande, S. S. and Raghavarao, D. (1964). Affine α-resolvable incomplete block designs. Contributions to Statistics, Volume presented to Professor P. C. Mahalnobis on his 75th birthday, Pergamon Press, Oxford and Statistical Publishing Society, Calcutta, 471–480.
65. Shyam S, (2023). Some methods of constructions of Partially Balanced Designs. Journal of Statistics and Computer Science, Vol. 2, pp. 47-53.
66. Singh, G. (2013). A new method of constructing SRGD & Resolvable SRGD designs. Mathematical Theory and Modelling, Vol. 3, 103-105.
67. Sinha, K. (2008). Optimal group divisible designs from affine resolvable designs. Statistics and Applications, Vol. 6, No. 1 & 2, 171-174.
68. Van Trung, T. (1999) Constructions for near resolvable designs and BIBDs. Journal of Combinatorial Designs, 7(3), 227-231.
69. Yadav, K., Dash, S., and Kumar A., (2024) International Journal of Statistics and Applied Mathematics; 9(4): 152-155.
70. Yan, Z. and Beiliang, D. (2004). α- Resolvable Group Divisible Designs with Block size three. Weily Interscience.
71. Yates, F. (1936). A new method of arranging variety trials involving a large number of varieties. Agr. Sci. 26, 424-55.
72. Yates, F. (1940). Lattice squares. J. Agricultural Sci. 30 672–687.